# **MCT333/ MCT344**: Mechatronic Systems Design
Lecture 4, 5, and 6: Actuator Selection and Sizing

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What is an Actuator?

❑ Actuators are devices used to produce action or motion.
❑ It is operated by a source of energy (mainly electrical signal, air ,fluids) and converts that energy into motion.
❑ Actuator is a mechanism by which a control system acts upon environment.
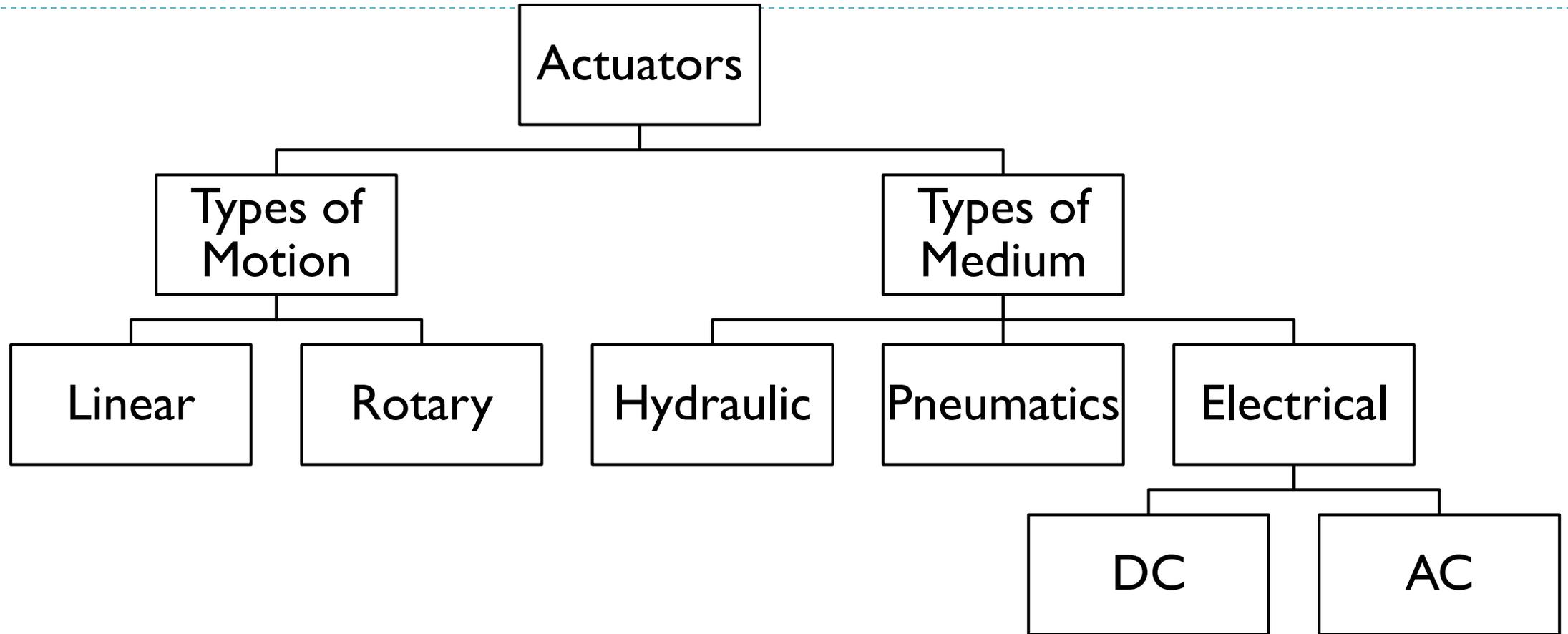❑ Actuator's output is usually Mechanical i.e. linear displacement or velocity.
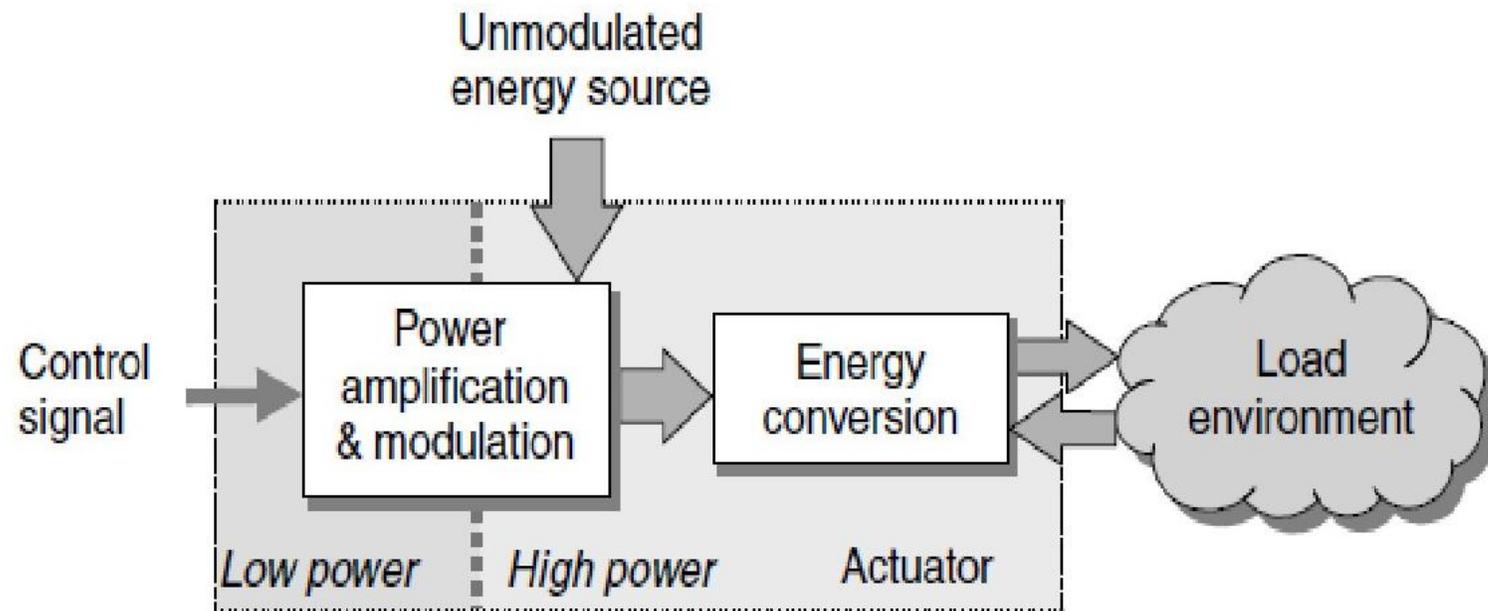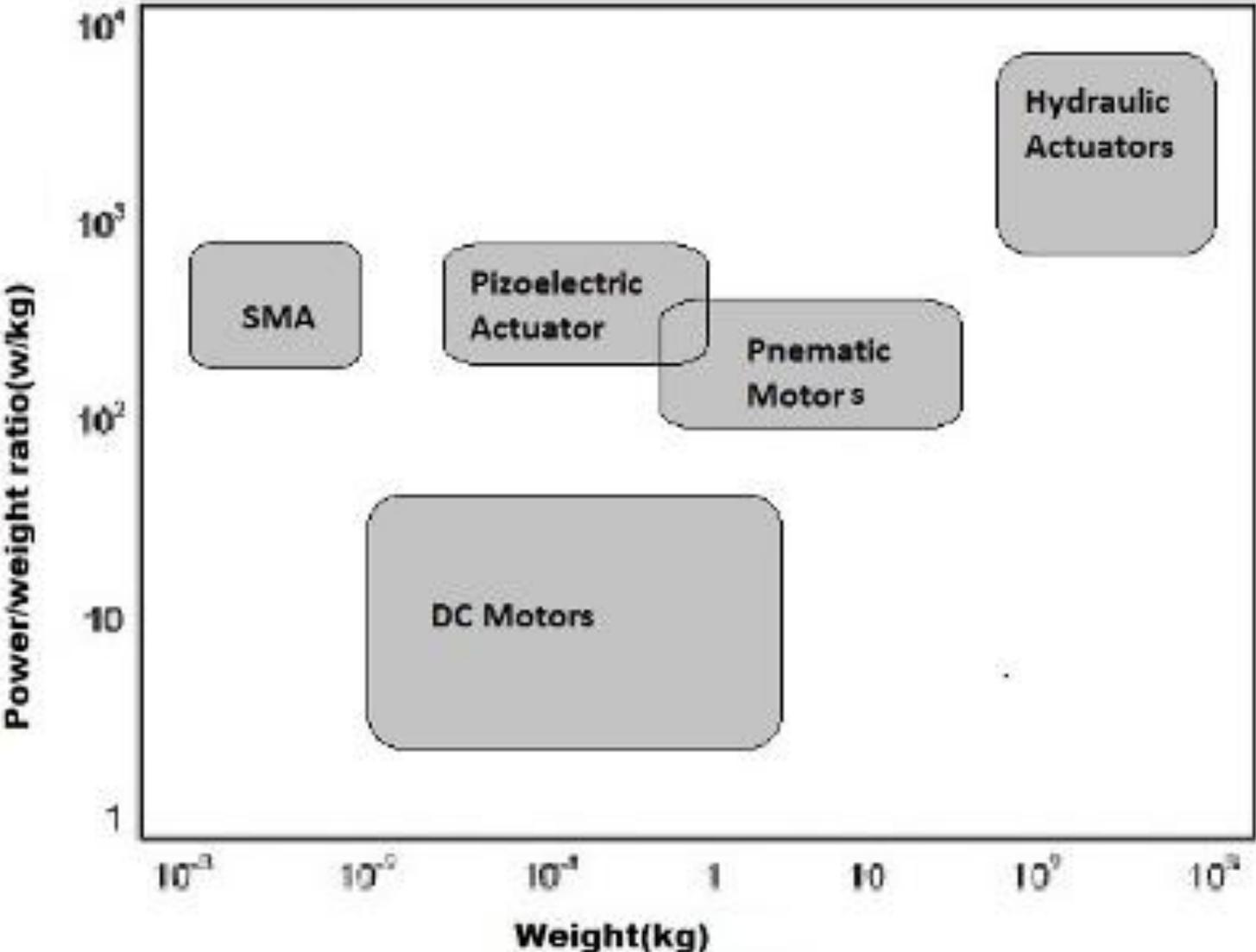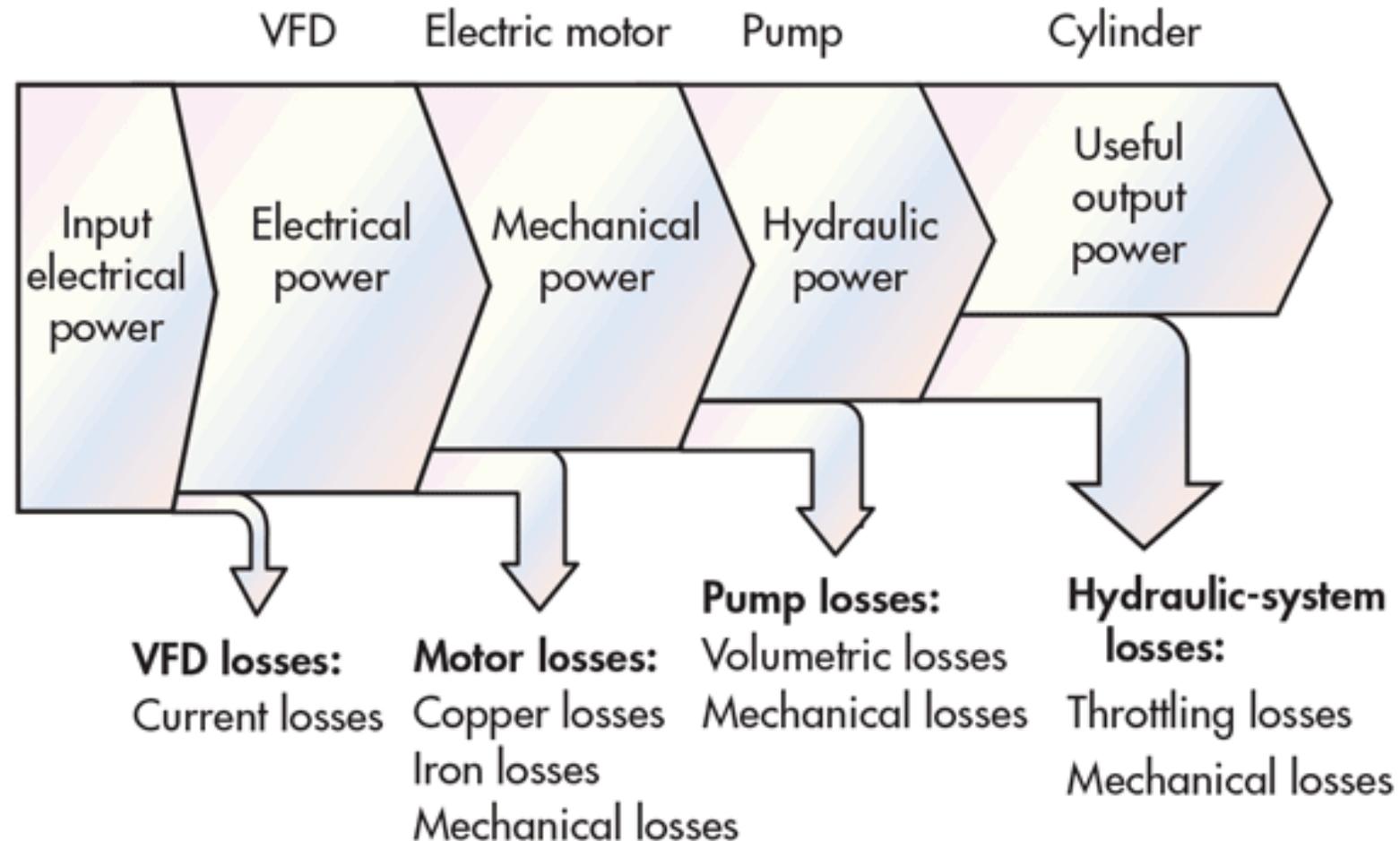
Actuator Function Diagram

Actuator Types?

```
                        ┌──────────────┐
                        │  Actuators   │
                        └──────┬───────┘
              ┌────────────────┴──────────────────┐
      ┌───────┴────────┐                  ┌────────┴────────┐
      │    Types of    │                  │    Types of     │
      │     Motion     │                  │     Medium      │
      └───────┬────────┘                  └────────┬────────┘
        ┌─────┴─────┐              ┌───────────────┼───────────────┐
   ┌────┴───┐  ┌────┴───┐    ┌─────┴────┐   ┌──────┴─────┐   ┌──────┴─────┐
   │ Linear │  │ Rotary │    │ Hydraulic│   │ Pneumatics │   │ Electrical │
   └────────┘  └────────┘    └──────────┘   └────────────┘   └──────┬─────┘
                                                              ┌──────┴─────┐
                                                         ┌────┴───┐  ┌────┴───┐
                                                         │   DC   │  │   AC   │
                                                         └────────┘  └────────┘
```

Actuators Power-to-weight Ratio

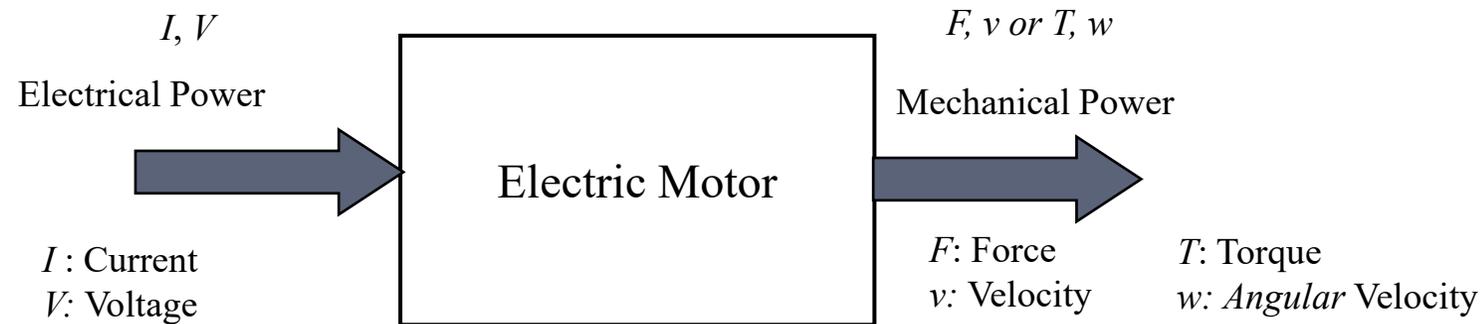# Operational Efficiency and Energy Conversion (Hydraulic System)

Analysis of Electric Motors

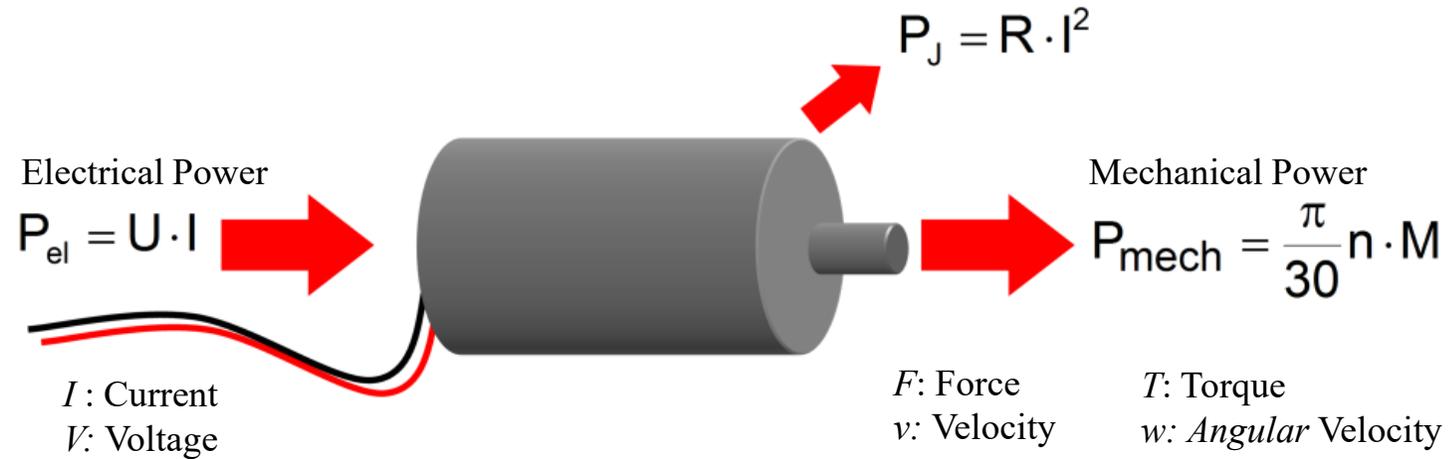▸ Electric Motors convert electrical power to mechanical power.

I, V

Electrical Power

F, v or T, w

Mechanical Power

Electric Motor

I : Current
V: Voltage

F: Force
v: Velocity

T: Torque
w: Angular Velocity

$Electrical\ Power = I*V$
$Mechanical\ Power = F*v$ for linear motor
$\qquad\qquad\quad = T*\omega$ for rotary motor

Electric Motors

Electric Motors convert electrical power to mechanical power.

$$P_J = R \cdot I^2$$

Electrical Power

$$P_{el} = U \cdot I$$

Mechanical Power

$$P_{mech} = \frac{\pi}{30} n \cdot M$$

I : Current
V: Voltage
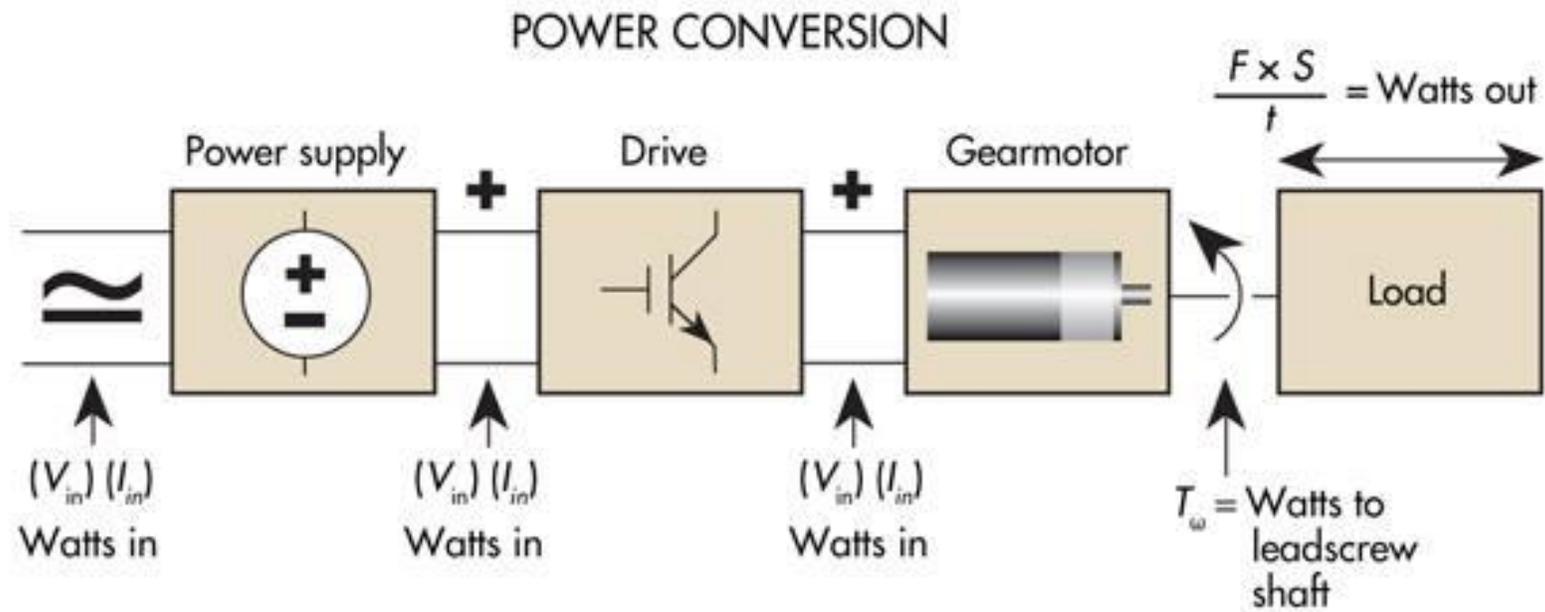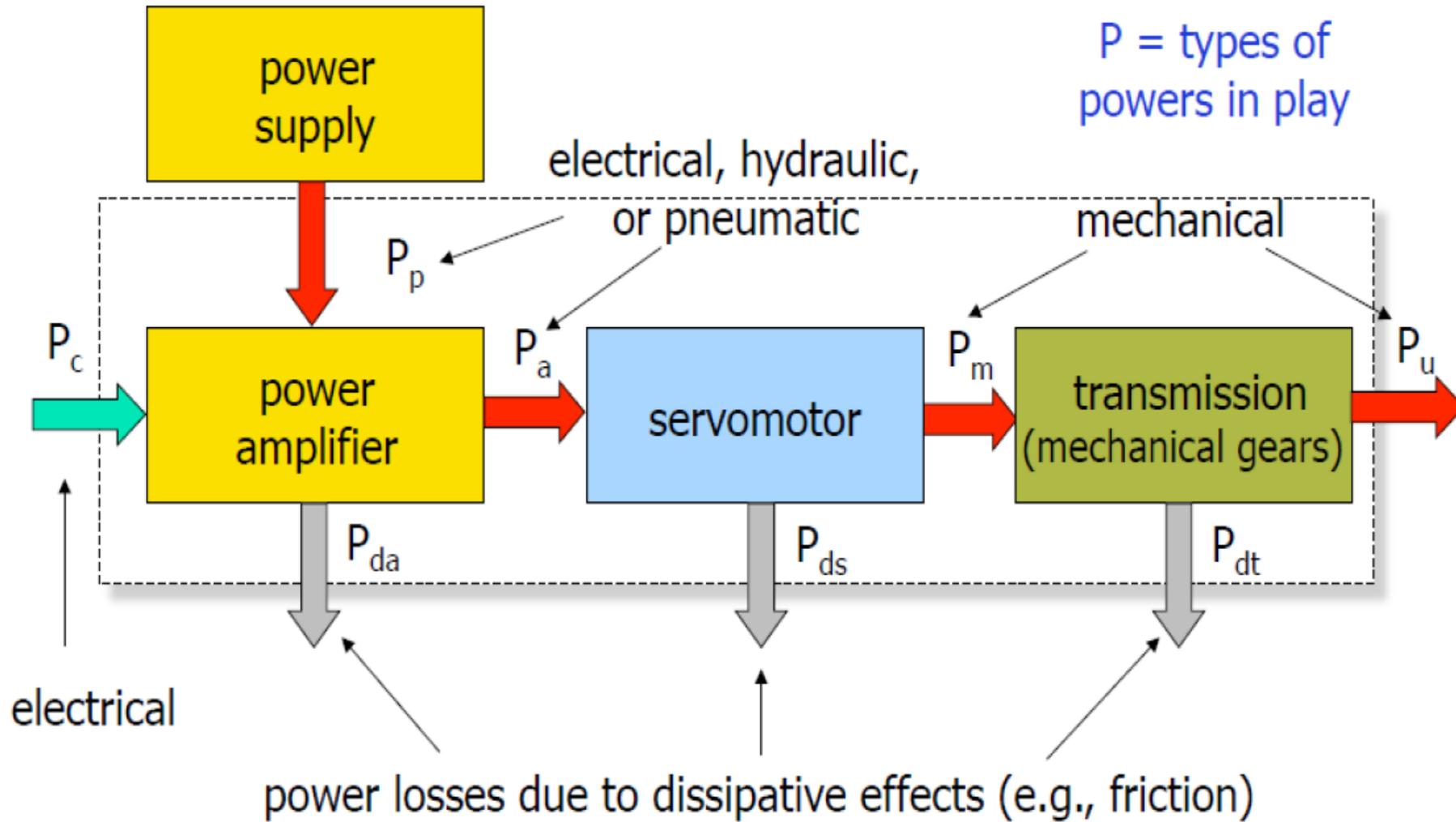
F: Force T: Torque
v: Velocity w: *Angular* Velocity

Electrical Power = I*V
Mechanical Power = F*v for linear motor
= T*ω for rotary motor

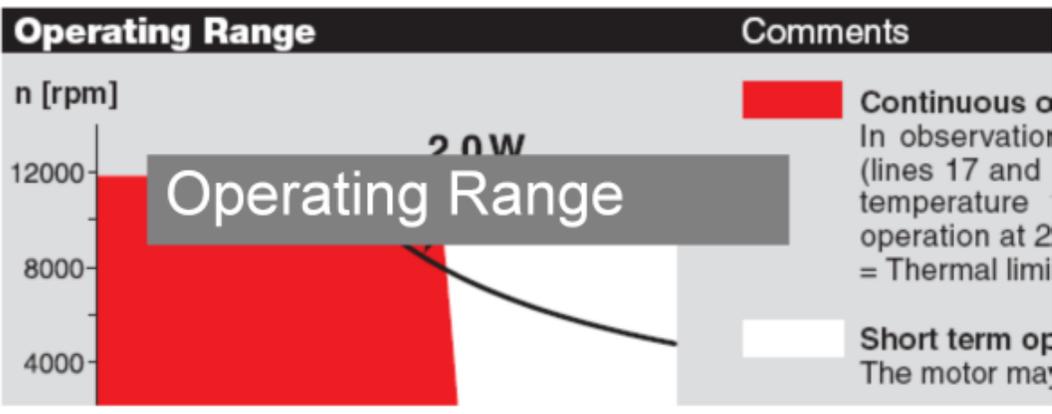# Operational Efficiency and Energy Conversion (Electrical Motor Actuation System)

POWER CONVERSION

Data Sheet and Operating Ranges

			110061	110062	**110063**	110064	**110065**	110066	110067	110068	**110069**	110070
Motor Data												
	Values at nominal voltage											
1	Nominal voltage	V	1.5	3.0	6.0	9.0	12.0	14.0	15.0	18.0	21.0	30.0
2	No load speed	rpm	10200	11700	9620	11800	11800	11800	11200	11200	11600	10800
3	No load current	mA	201	117	46.7	39.1	29.3	25.1	22.2	18.5	16.5	10.7
4	Nominal speed	rpm	867								70	4160
5	Nominal torque (max. continuous torque)	mNm	0.68								39	2.35
6	Nominal current (max. continuous current)	A	0.720	0.720	0.494	0.394	0.294	0.253	0.225	0.186	0.162	0.105
7	Stall torque	mNm	4.93	4.51	4.02	4.82	4.76	4.81	4.53	4.47	4.48	4.03
8	Starting current	A	3.76	1.97	0.721	0.700	0.519	0.450	0.377	0.310	0.275	0.164
9	Max. efficiency	%	58	57	56	58	58	58	58	57	57	55
	Characteristics											
10	Terminal resistance	Ω	0.399	1.52	8.32	12.8	23.1	31.1	39.8	58.0	76.2	183
11	Terminal inductance	mH	0.017	0.0519	0.306	0.467	0.831	1.13	1.42	2.05	2.61	6.01
12	Torque constant	mNm / A	1.31								.3	24.7
13	Speed constant	rpm / V	729								37	387
14	Speed / torque gradient	rpm / mNm	2220	2770	2560	2600	2630	2600	2630	2670	2750	2880
15	Mechanical time constant	ms	24.5	23.7	23.2	23.2	23.2	23.2	23.4	23.3	23.4	23.8
16	Rotor inertia	gcm²	1.05	0.816	0.864	0.854	0.844	0.854	0.848	0.834	0.811	0.788

Values at nominal voltage

Characteristic motor data

Specifications

Thermal data

17	Thermal resistance housing-ambient	29.8 K / W
18	Thermal resistance winding-housing	5.5 K / W
19		3 s
20		3 s
21	Ambient temperature	-30 ... +85°C
22	Max. permissible winding temperature	+125°C

Mechanical data (sleeve bearings)

23	Max. permissible speed	11900 rpm

Further Specifications

Operating Range

Comments

n [rpm]

2.0 W

12000

8000

4000

Operating Range

- Continuous o
  In observation
  (lines 17 and
  temperature
  operation at 2
  = Thermal limi
- Short term op
  The motor ma

EC-4pole 30 ⌀30 mm, brushless, 200 Watt

High Power

M1.6x2.4 tief/deep

⊕ ⌀0.2 B

3x 120°

20.9

⊥ ⌀0.1 C

↗ 0.02

A

C

⌀16 0/-0.008
⌀5 0/-0.004

1.2 0/-0.1

2.1 0/-0.2

⌀30 0/-0.25

20 0/-0.2

64 max.

⌀4 0/-0.004
⌀5 0/-0.004

B

11.5 +0.2/0

14.5 0/-0.7

⌀16 0/-0.05

300 ±20

⊕ ⌀0.2 A

M3x4.5 tief/deep

4x 90°

22

M 1:2
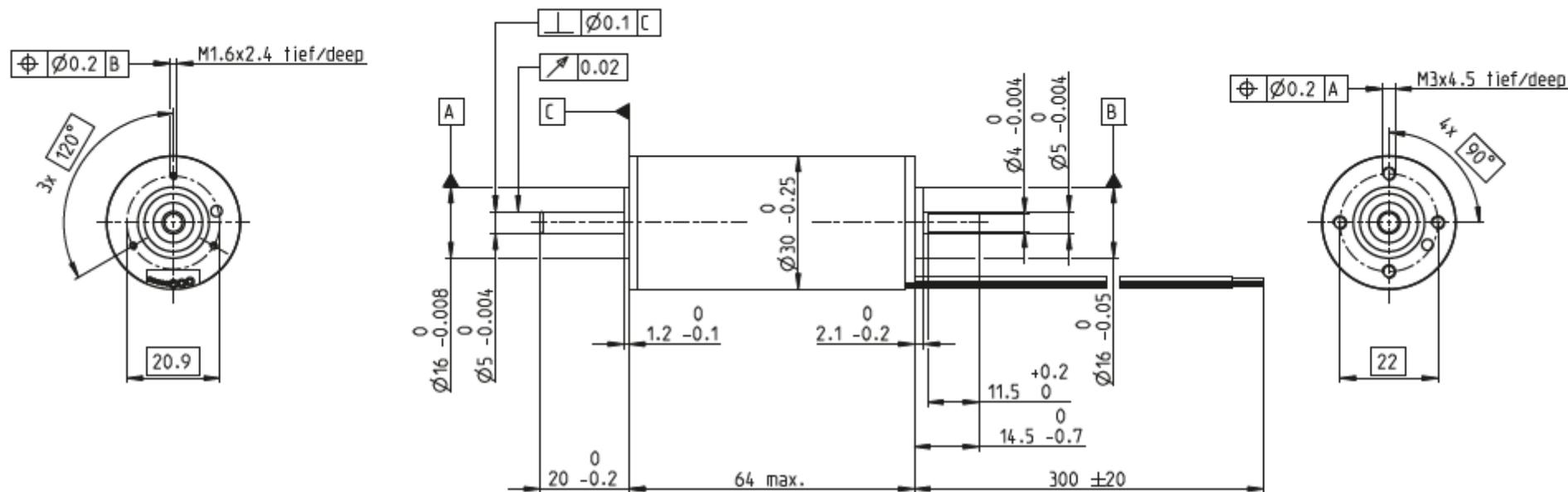
Stock program
Standard program
Special program (on request)

Part Numbers								
	305013	305014	**305015**					
Motor Data								
Values at nominal voltage								
1 Nominal voltage	V	24	36	48				
2 No load speed	rpm	16700	16700	16500				
3 No load current	mA	728	485	356				
4 Nominal speed	rpm	16100	16200	16000				
5 Nominal torque (max. continuous torque)	mNm	94.6	94.2	92.9				
6 Nominal current (max. continuous current)	A	7.58	5.03	3.68				

Stock program
Standard program
Special program (on request)

Motor Data		305013	305014	305015				
Values at nominal voltage								
1 Nominal voltage	V	24	36	48				
2 No load speed	rpm	16700	16700	16500				
3 No load current	mA	728	485	356				
4 Nominal speed	rpm	16100	16200	16000				
5 Nominal torque (max. continuous torque)	mNm	94.6	94.2	92.9				
6 Nominal current (max. continuous current)	A	7.58	5.03	3.68				
7 Stall torque	mNm	3220	3510	3430				
8 Stall current	A	236	171	124				
9 Max. efficiency	%	89	90	90				
Characteristics								
10 Terminal resistance phase to phase	Ω	0.102	0.21	0.386				
11 Terminal inductance phase to phase	mH	0.016	0.037	0.065				
12 Torque constant	mNm/A	13.6	20.5	27.6				
13 Speed constant	rpm/V	700	466	346				
14 Speed/torque gradient	rpm/mNm	5.21	4.78	4.83				
15 Mechanical time constant	ms	1.82	1.67	1.69				
16 Rotor inertia	gcm²	33.3	33.3	33.3				

Specifications

Thermal data
17	Thermal resistance housing-ambient	7.4 K/W
18	Thermal resistance winding-housing	0.21 K/W
19	Thermal time constant winding	2.11 s
20	Thermal time constant motor	1180 s
21	Ambient temperature	-20...+100°C
22	Max. winding temperature	+155°C

Mechanical data (preloaded ball bearings)
23	Max. speed	25 000 rpm
24	Axial play at axial load < 8.0 N	0 mm
	> 8.0 N	0.14 mm
25	Radial play	preloaded
26	Max. axial load (dynamic)	5.5 N
27	Max. force for press fits (static)	73 N
	(static, shaft supported)	1300 N
28	Max. radial load, 5 mm from flange	25 N

Other specifications
29	Number of pole pairs	2
30	Number of phases	3
31	Weight of motor	300 g

Values listed in the table are nominal.

Connection motor (Cable AWG 18)
black	Motor winding 2
white	Motor winding 3
red	Motor winding 1

Connection sensors (Cable AWG 26)
black/grey	Hall sensor 2
blue	GND
green	V_{Hall} 3...24 VDC
red/grey	Hall sensor 1
white/grey	Hall sensor 3

Wiring diagram for Hall sensors see p. 35

Operating Range

Comments
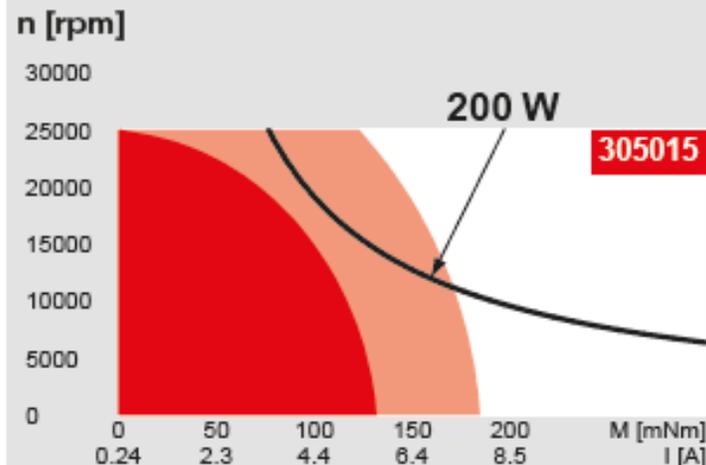
- Continuous operation
- Continuous operation with reduced thermal resistance R_{th2} 50%
- Intermittent operation
- Assigned power rating

maxon Modular System

Overview on page 20–27

Planetary Gearhead
∅32 mm
8 Nm
Page 345

Planetary Gearhead
∅42 mm
3 - 15 Nm
Page 350
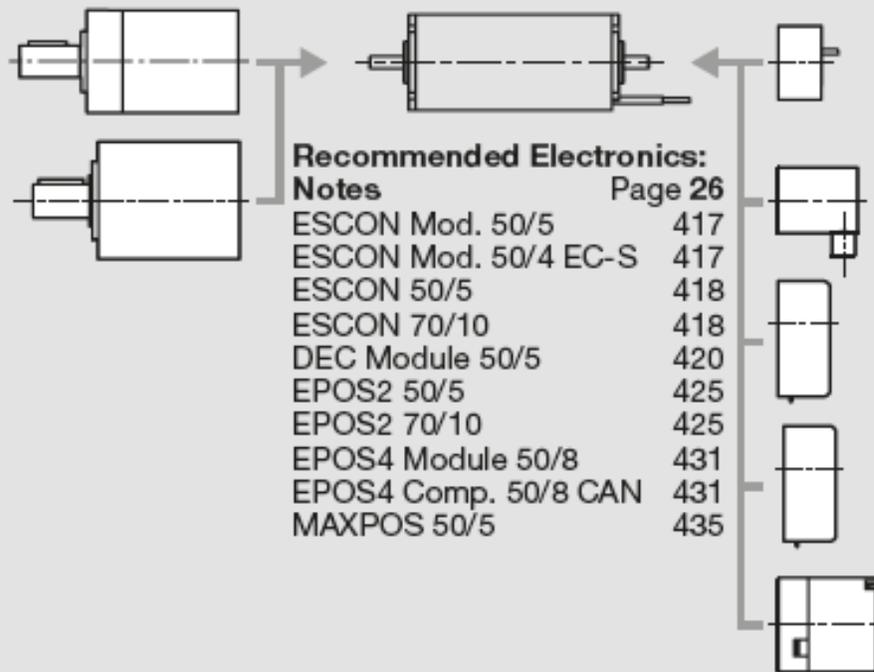
Recommended Electronics:
Notes	Page 26
ESCON Mod. 50/5	417
ESCON Mod. 50/4 EC-S	417
ESCON 50/5	418
ESCON 70/10	418
DEC Module 50/5	420
EPOS2 50/5	425
EPOS2 70/10	425
EPOS4 Module 50/8	431
EPOS4 Comp. 50/8 CAN	431
MAXPOS 50/5	435

Encoder MR
128 - 1000 CPT,
3 channels
Page 392

Encoder 2RMHF
3000 - 5000 CPT,
3 channels
Page 397

Encoder HEDL 5540
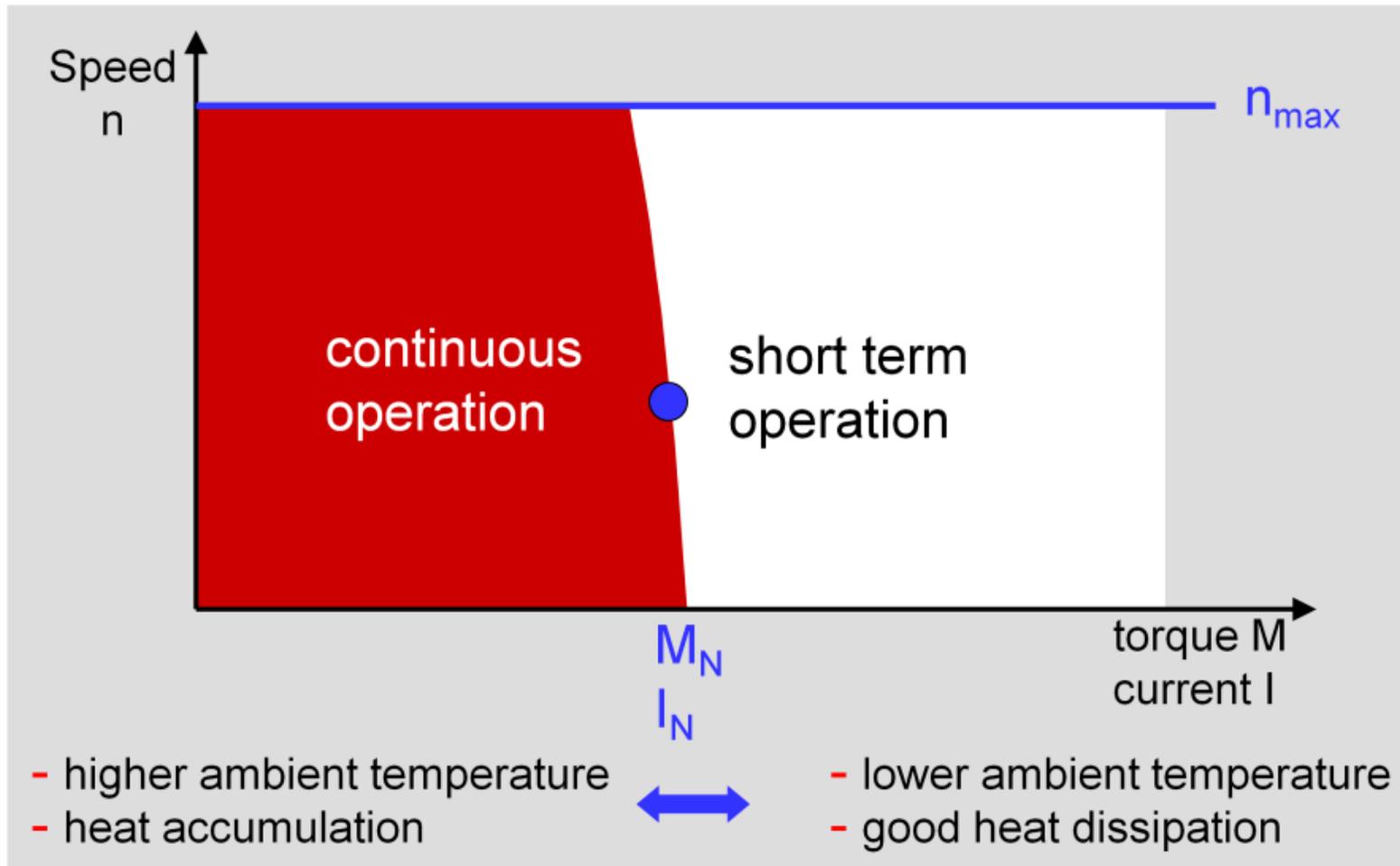500 CPT,
3 channels
Page 404

Encoder AEDL 5810
1024 - 5000 CPT,
3 channels
Page 407

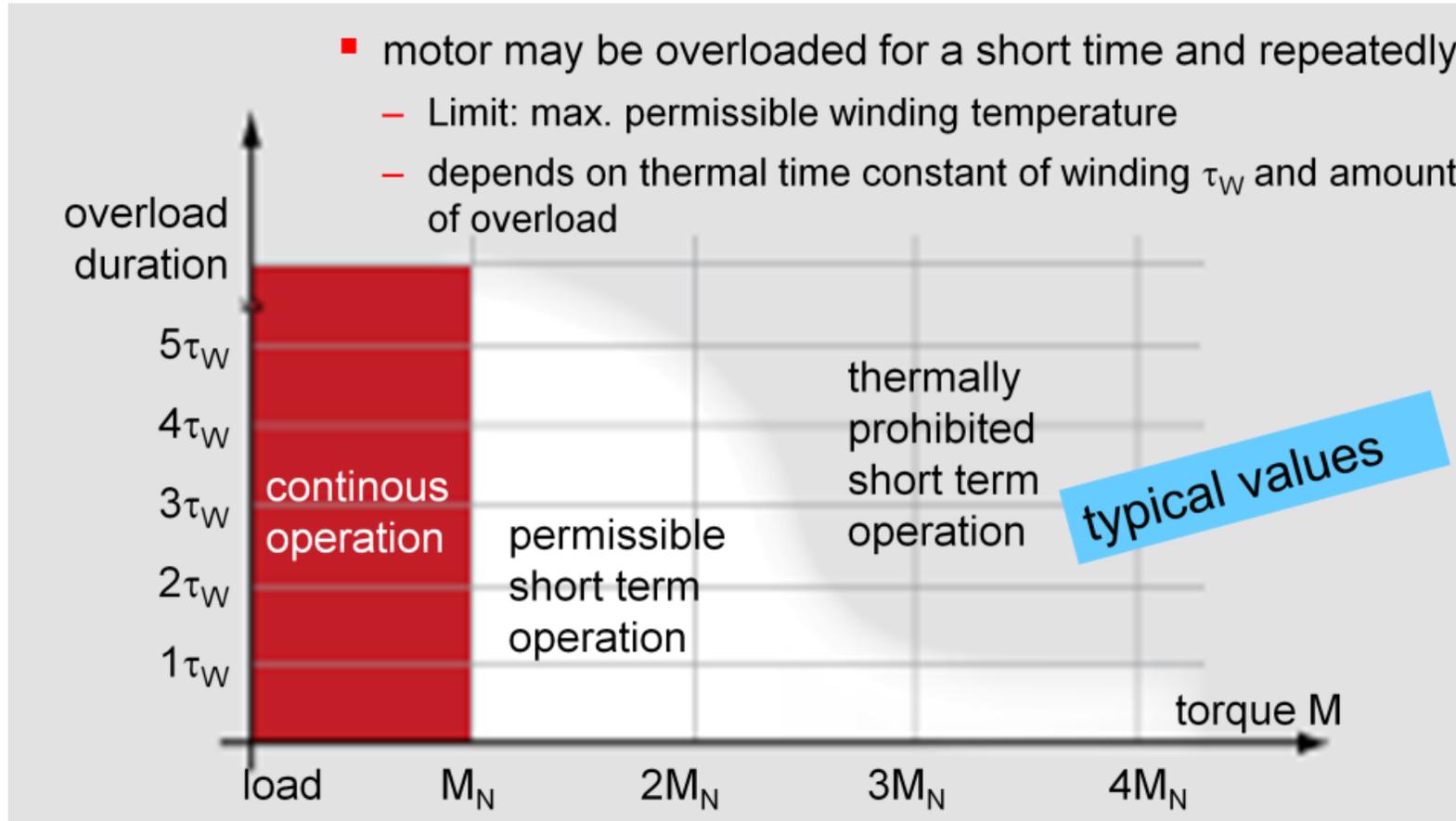**Brake AB 20**
24 VDC
0.1 Nm
Page 444

Operation Ranges

Motor limits: operation ranges

Speed n — vertical axis

n_{max}

continuous operation

short term operation

M_N
I_N

torque M
current I

- higher ambient temperature
- heat accumulation

- lower ambient temperature
- good heat dissipation

Operation Ranges

Short-term operation at overload

- motor may be overloaded for a short time and repeatedly
  - Limit: max. permissible winding temperature
  - depends on thermal time constant of winding τ_W and amount of overload

overload duration

$5\tau_W$
$4\tau_W$
$3\tau_W$ — continous operation
$2\tau_W$
$1\tau_W$

permissible short term operation

thermally prohibited short term operation

typical values

load M_N $2M_N$ $3M_N$ $4M_N$

torque M

thermal time constant of the winding τ_W.

Thermal motor data

Thermal motor data

describe the motor heating and thermal limits

- depend strongly on mounting conditions
- standard mounting:

- heating and cooling
  - thermal resistance housing-ambient R_{th2}
  - thermal resistance winding-housing R_{th1}
  - thermal time constant of winding τ_{thW}
  - thermal time constant of motor τ_{thS}
- temperature limits
  - ambient temperature range
  - max. winding temperature T_{max}

plastic plate

horizontal mounting
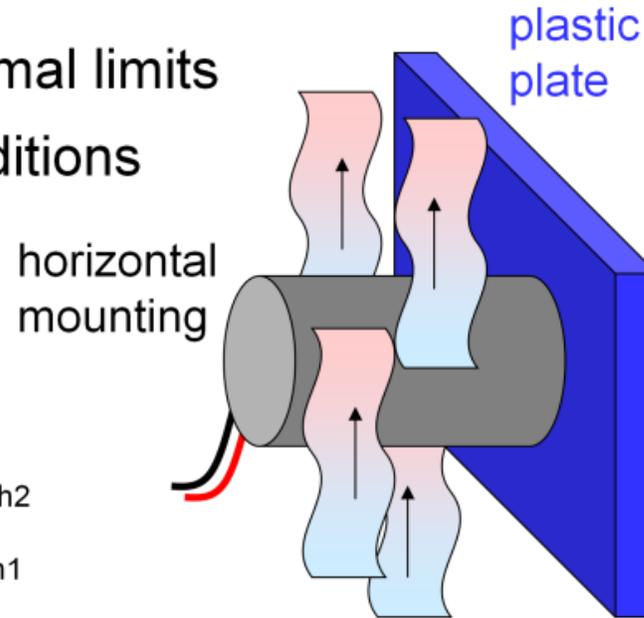
free convection at 25 °C ambient temperature

Cooling Fan
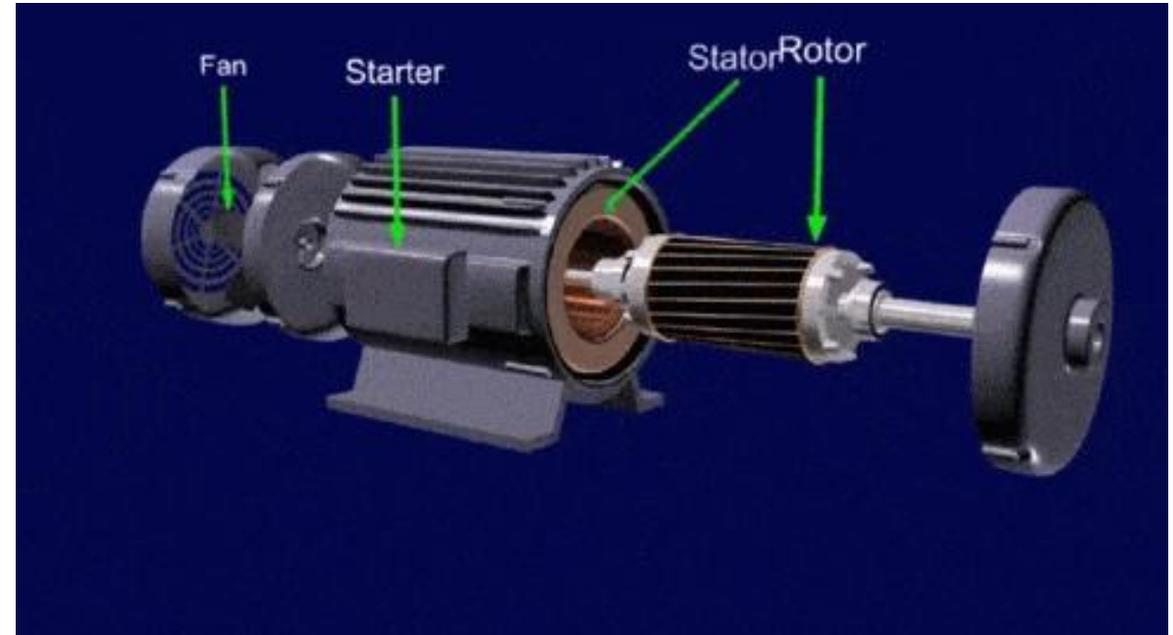
Mechanical Motor Data

Mechanical motor data

describe maximum speed and the properties of bearings
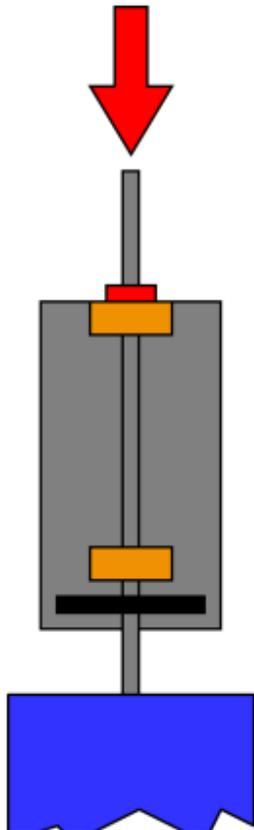
- **max. permissible speed**
  - limited by bearing life considerations (EC)
  - limited by relative speed between collector and brushes (DC)
- **axial and radial play**
  - suppressed by a preload
- **axial and radial bearing load**
  - dynamic: in operation
  - static: at stall

What is servomotor?

A **servomotor** is a rotary actuator or linear actuator that allows for precise control of angular or linear position, velocity and acceleration. It consists of a suitable **motor** coupled to a sensor for position feedback.
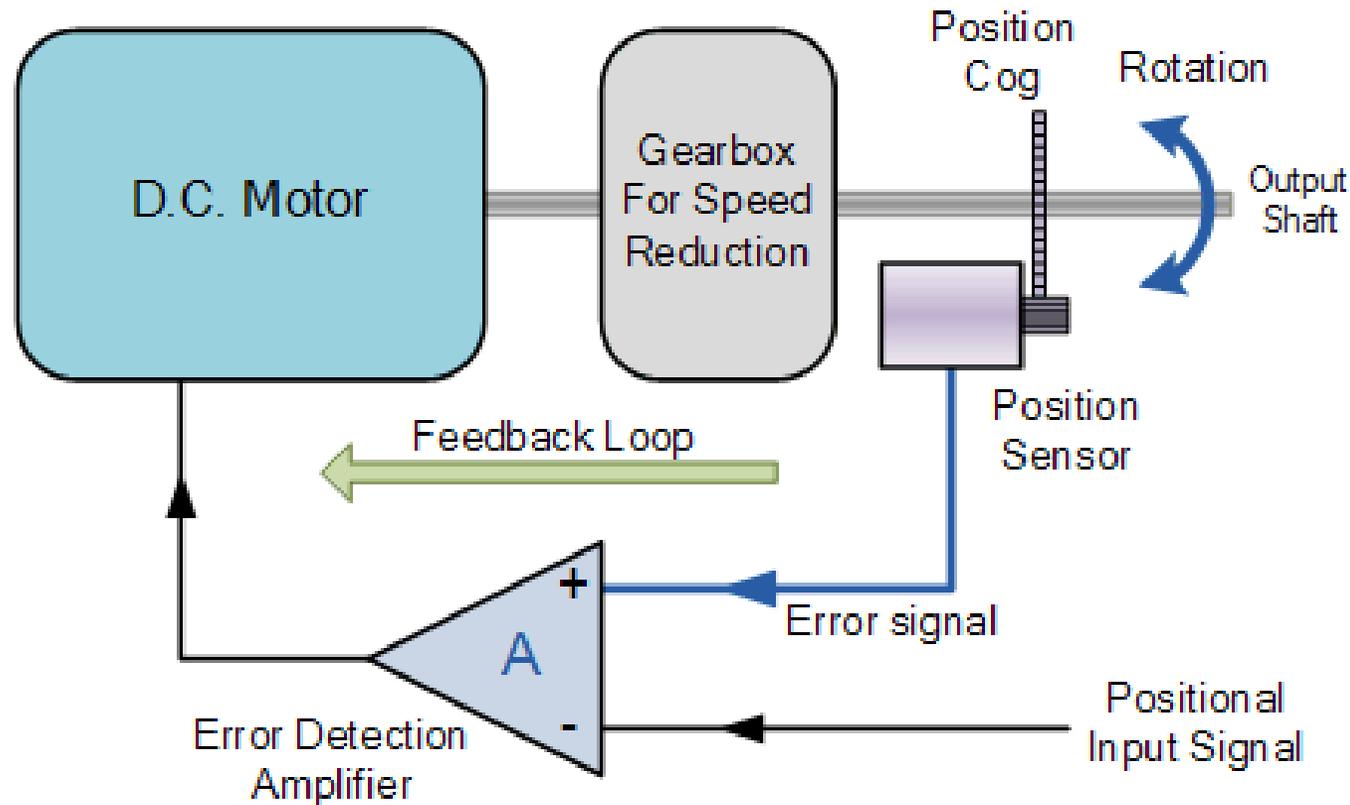
Operating Modes (Four Quadrant)
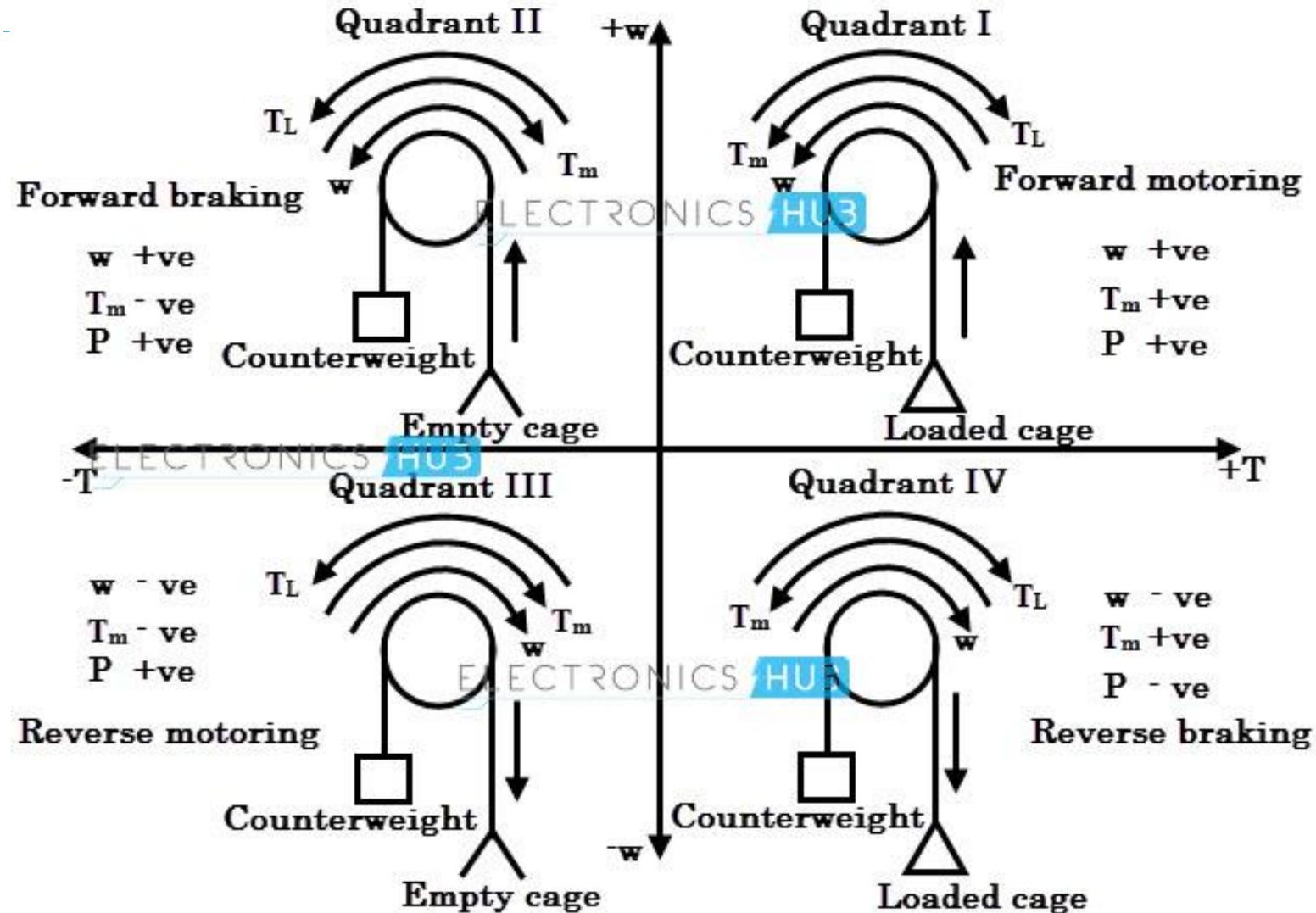
Actuator sizing

Types of Motion and Motion Conversion

Linear and Angular Motion

The linear motion induced in a rigid object is governed by Newton's second law of motion

$F = ma$

F is the resultant of all forces acting on the object, m is the mass of the object and a is the resulting linear acceleration.

The constant force F produces a constant acceleration a and moves the object of mass m a certain distance s according to

$s = 1/2\ at^2$

s is the displacement and t is the time

Thus, the time required to move mass m through distance s by means of a constant force F is given by $t = \sqrt{2ms/F}$

▶

For angular motion, Newton's law reads

$$T = J \ddot{\theta}$$

T is the resultant of all torques acting on a mass rotating about a fixed axis, J is the moment of inertia of the mass about its axis of rotation and $\ddot{\theta}$ is the angular acceleration
and the angular displacement equation analogous to that of linear motion is *$\theta = 1/2\ at^2$*

θ *is the angular displacement.*

Solving for *t yields .* $\ t = \sqrt{2J\theta/T}$

The Move Profile

Typical Move Profile (Figure 6)
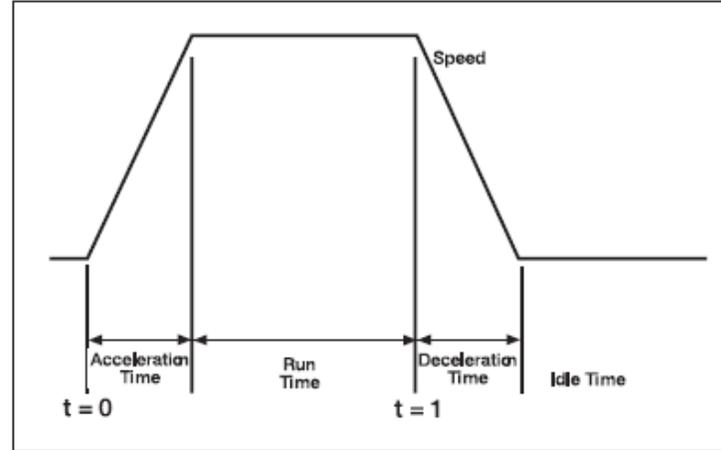
Example Move Profile (Figure 7)

Motion Profile
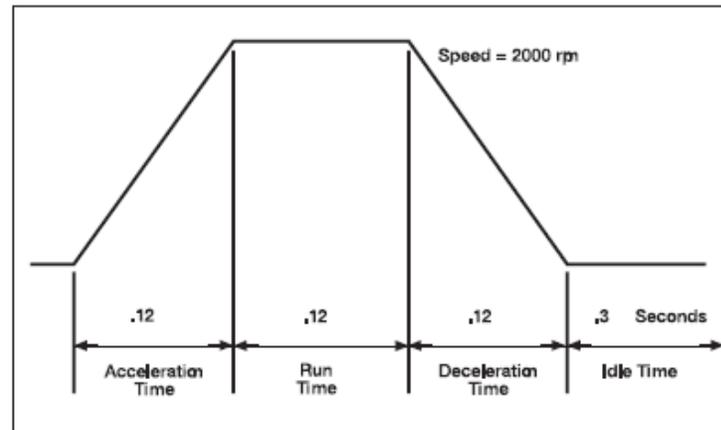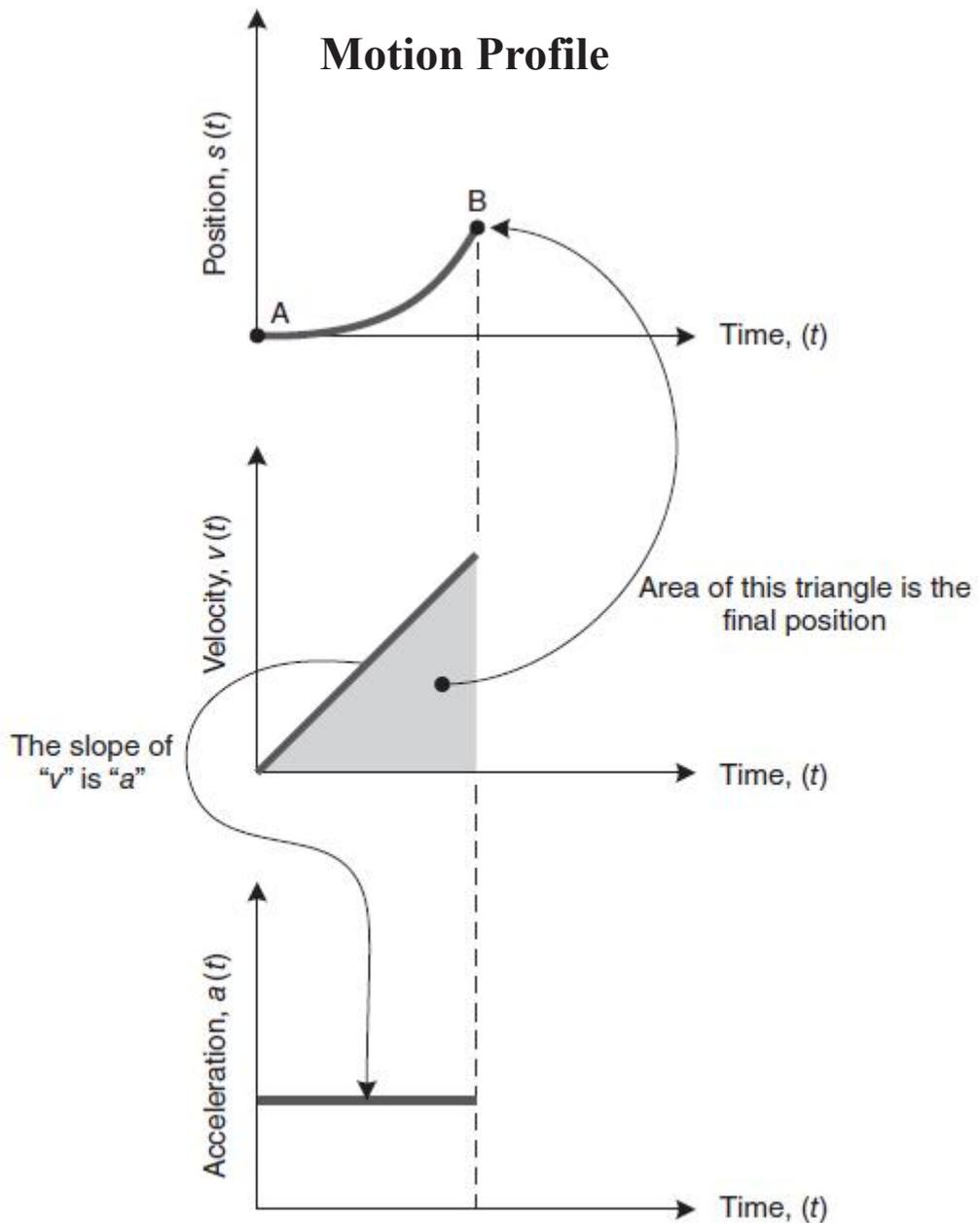
Figure 2.1 Basic relationships between position, velocity, and acceleration

Position, s (t)

B

A

Time, (t)

Velocity, v (t)

Area of this triangle is the final position

The slope of "v" is "a"

Time, (t)

Acceleration, a (t)

Time, (t)

Position

s

25

Time, (t)

Velocity

10

5

15

Time, (t)

Acceleration

a

Time, (t)
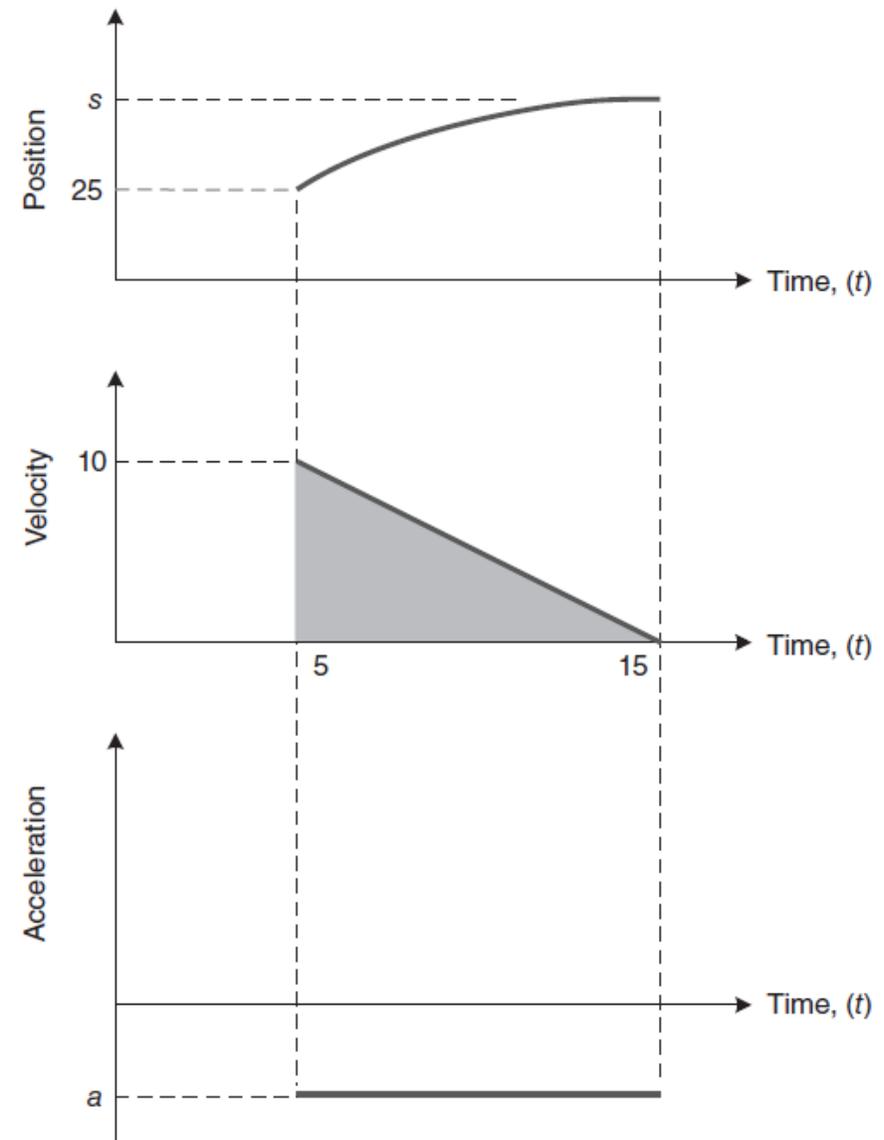
Figure 2.3 Velocity profile for deceleration

Trapezoidal Velocity Profile

Position, s(t)

Velocity, v(t)

Acceleration, a(t)

Jerk, j(t)

Infinite

t_a t_m t_d

t_{total}
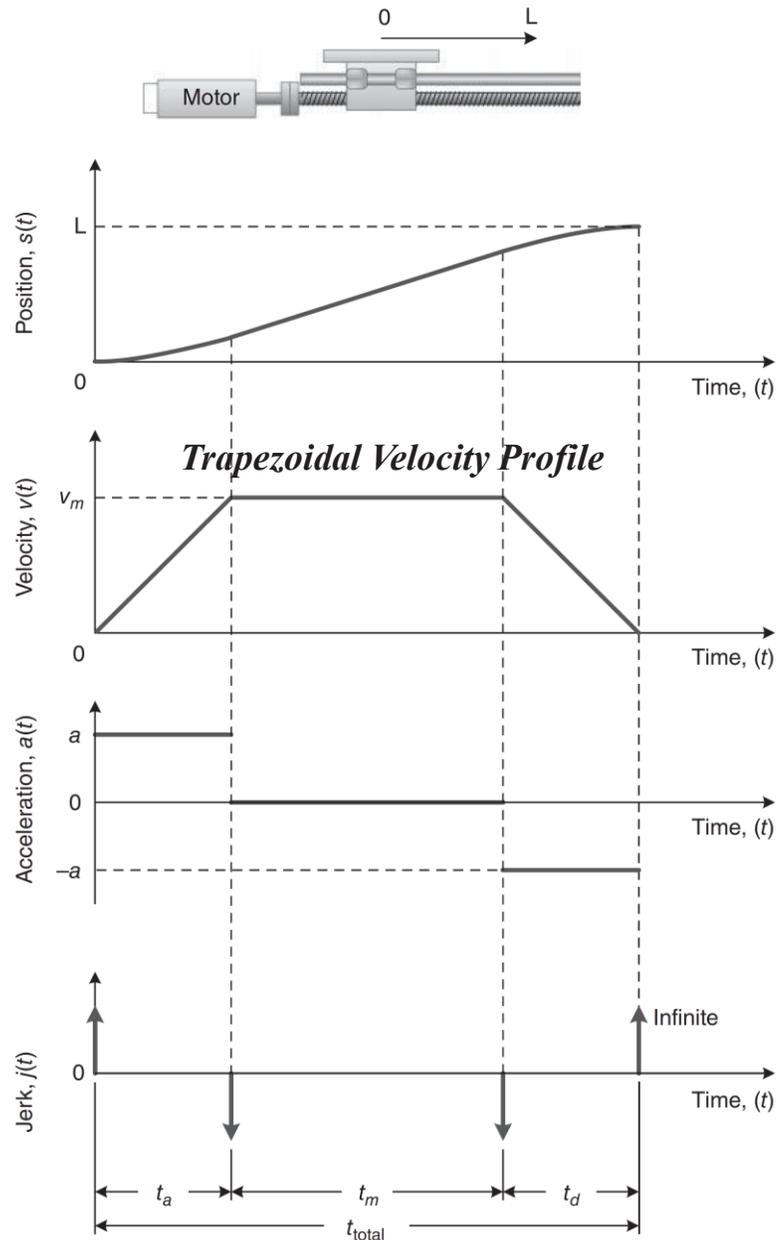
Figure 2.4 Trapezoidal velocity profile and associated position, acceleration, and jerk profiles to move an axis from 0 to position L

S-curve Velocity Profile

Position, s(t)

Velocity, v(t)

$v(t) = C_1 t^2 + C_2 t + C_3$

Acceleration, a(t)

Jerk, j(t)

t_a t_m t_d

t_{total}
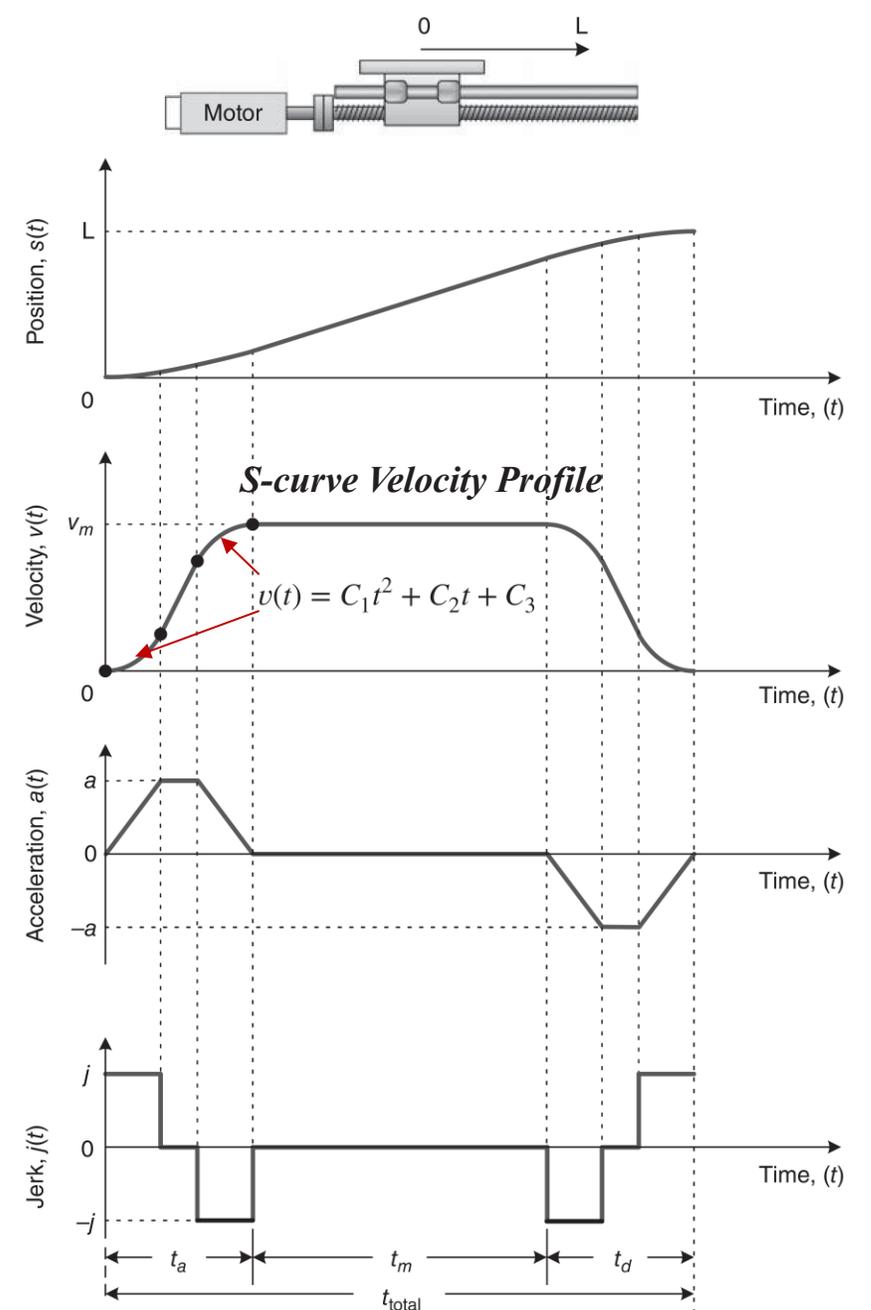
Figure 2.8 S-curve velocity profile and the associated position, acceleration, and jerk profiles

Example:

Consider a rotary motion axis driven by an electric servo motor. The rotary load is directly connected to the motor shaft without any gear reducer (Fig. 1). The rotary load is a solid cylindrical shape made of steel material, $d=75mm$, $l=50mm$, $\rho=7800kg/m^3$. The desired motion of the load is a periodic motion (Fig. 2).
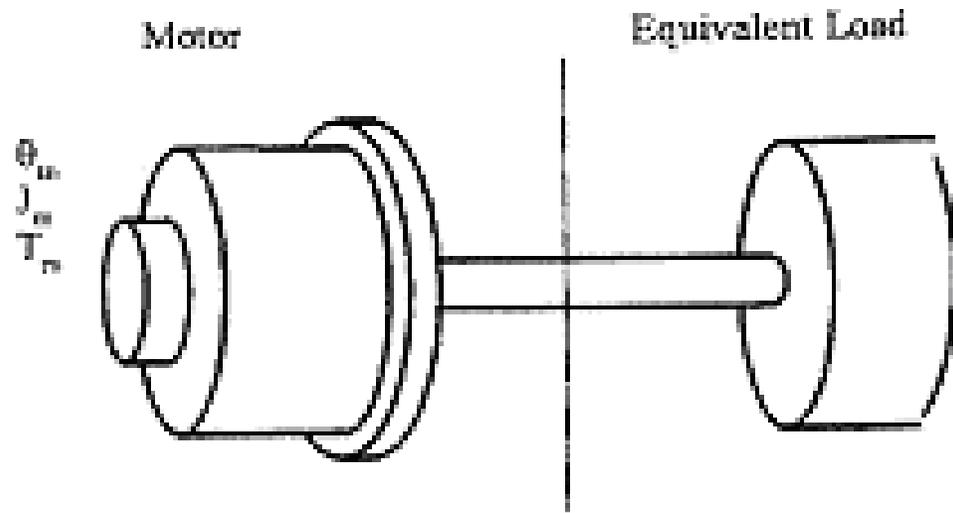
Fig. (1)
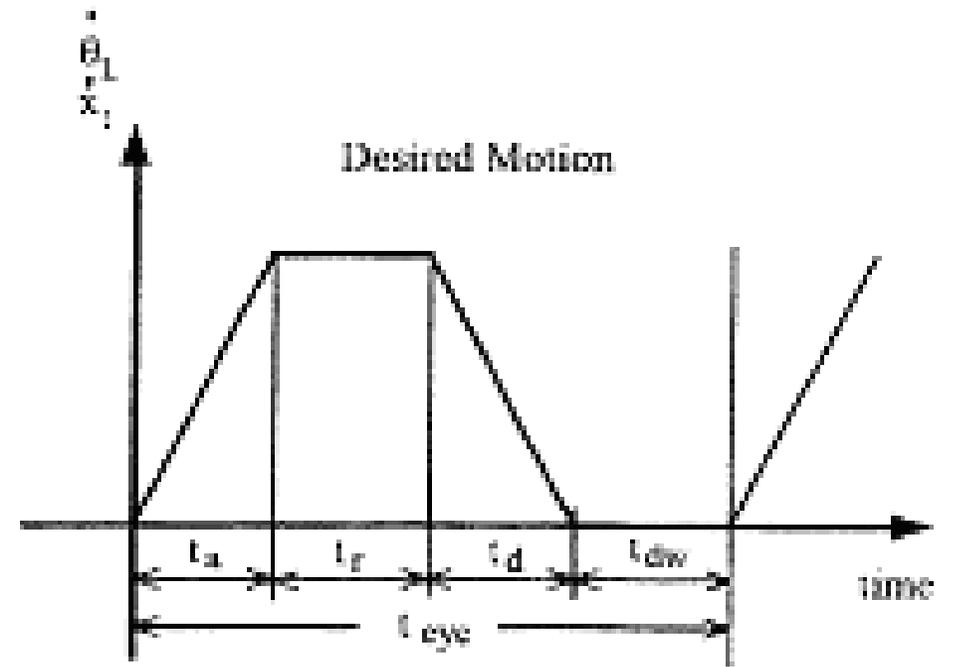
Fig.(2)

Example Cont.:

The total distance to be traveled is _1/4_ of a revolution. _The period of motion_ is t_{cyc}=250 _msec._, and dwell portion of it is t_{dw}=100 _msec._, and the remaining part of the cycle time is equally divided between acceleration, constant speed and deceleration periods, t_a=t_r=t_d=50 _msec._ _Determine the required motor size for this application_.
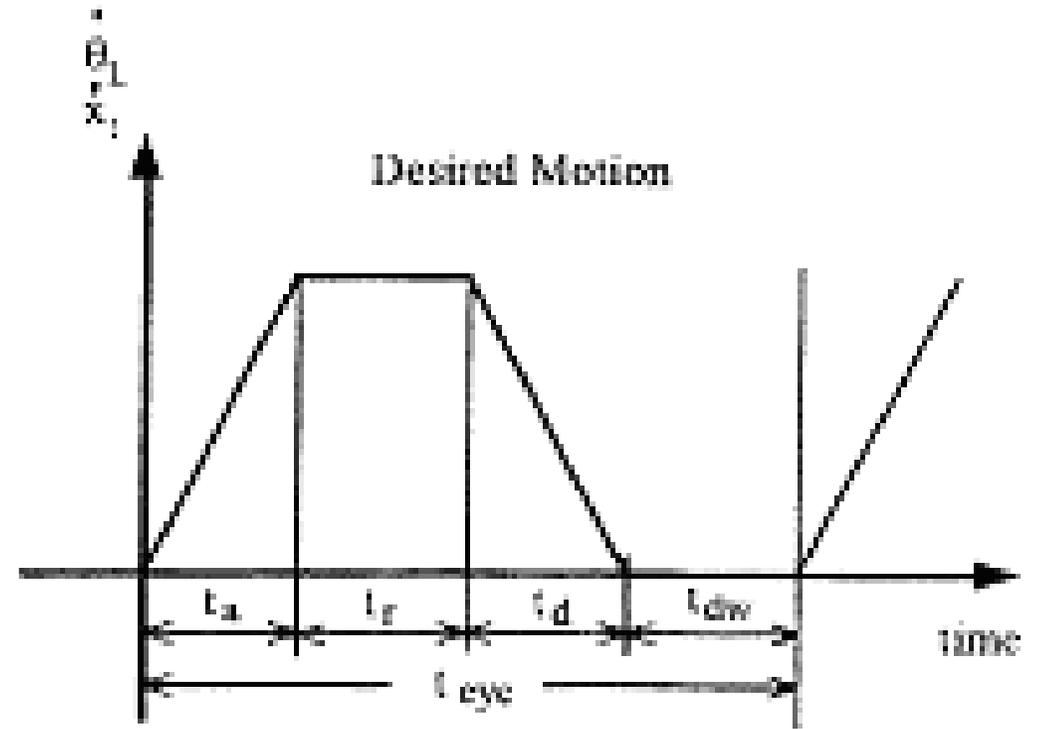
Fig.(2)
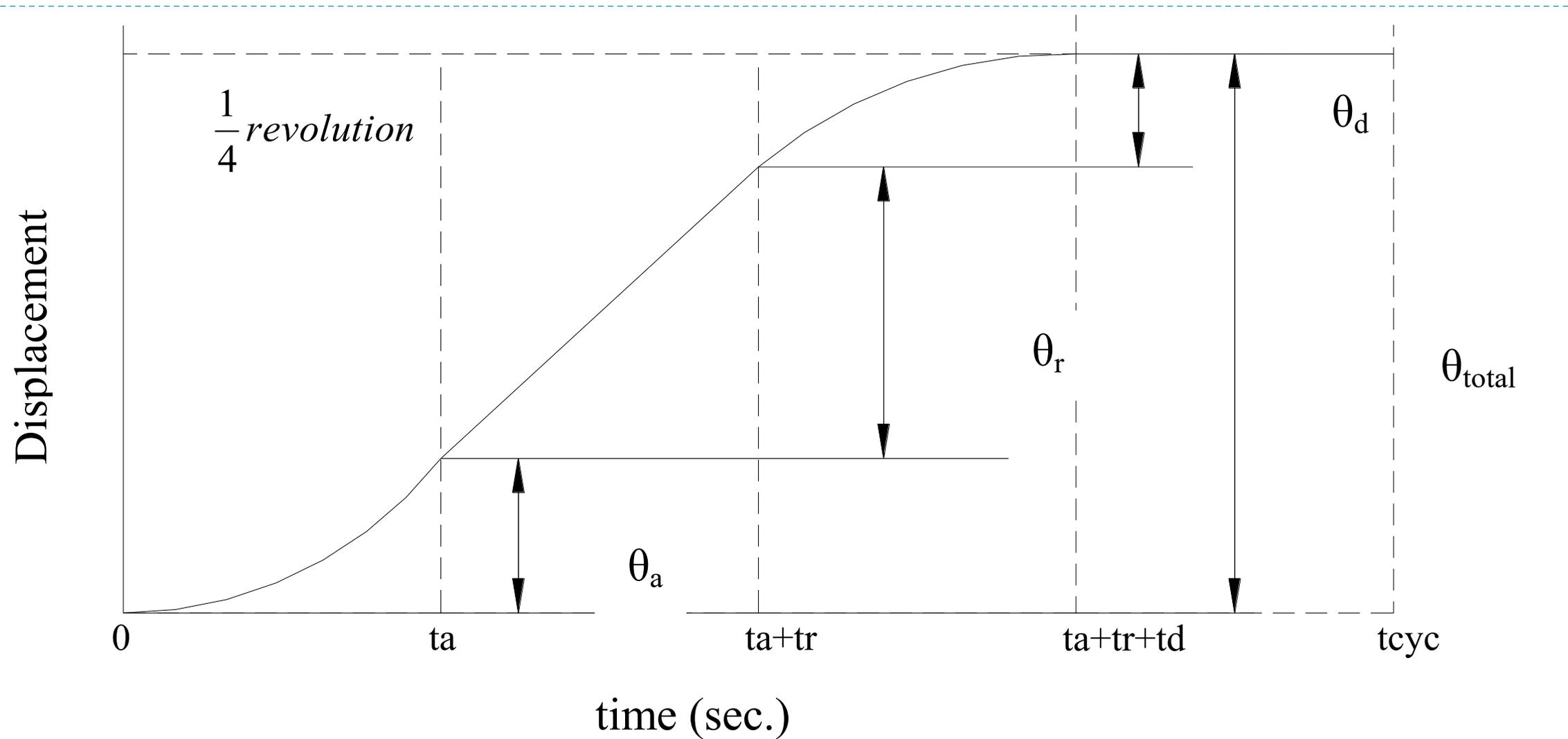
Actuator Sizing Algorithm:

1. Define the **geometric relationship** between the actuator and load. **In other words**, select the type of motion transmission mechanism between the motor and load (N=reduction ratio).
2. Define the inertia and torque\force characteristics of the load and transmission mechanisms, i.e. define **the inertia of the tool as well as the inertia of the gear reducer mechanisms** (J_l, T_l).
3. Define the **desired cyclic motion profile** in the load speed versus time ($\vartheta'_l(t)$).
4. Using the reflection equations developed above, **calculate the reflected load inertia and torque/force** (J_{eff}, T_{eff}) that will effectively act on the actuator shaft as well as the desired motion at the actuator shaft ($\vartheta'_m(t)$).
5. **Guess** a actuator/motor inertia from an available list (catalog) (**or** make the first calculation with zero motor inertia **assumption**), and calculate the torque history, $T_m(t)$, for the desired motion cycle. Then calculate the peak torque and RMS torque from $T_m(t)$.

Actuator Sizing Algorithm Cont.:

6. Check if the actuator size meets the required performance in terms of **peak and RMS** torque, and maximum speed capacity (T_p, T_{rms}, ϑ'_{max}). If the above selected actuator/motor from the available list does not meet the requirements (i.e. too small or too large), **repeat the previous step by selecting a different motor**. It should be noted that if a stepper motor is used, the torque capacity of the **stepper motor is rated only in terms of the continuous rating, not peak.** Therefore, the required peak and RMS torque must be smaller than the continuous torque capacity of the step motor.

7. Most servo motor continuous torque capacity rating is given for **25ºC ambient temperature** is different than 25°C, the continuous (RMS) torque capacity of the motor should be de-rated using the following equation for a temperature,

$$T_{rms} = T_{rms}(25^o C)\sqrt{\frac{(155 - Temp^o C)}{130}}$$

Solution:

1) Determine the Net inertia:

where:

J_{total}=the total inertia reflected on the motor axis.
J_{eff}=the load inertias reflected on the motor shaft➜ $J_{eff} = J_L$
J_m=the motor rotor inertia.

$$J_{eff} = \frac{1}{2}mr^2 = \frac{1}{2}(\rho(\frac{\pi d^2}{4})l)r^2$$

$$= \frac{1}{2} \times 7800 \times (\frac{\pi}{4} \times (75 \times 10^{-3})^2 \times 50 \times 10^{-3}) \times (\frac{75 \times 10^{-3}}{2})^2 = 0.0012 kg.m^2$$
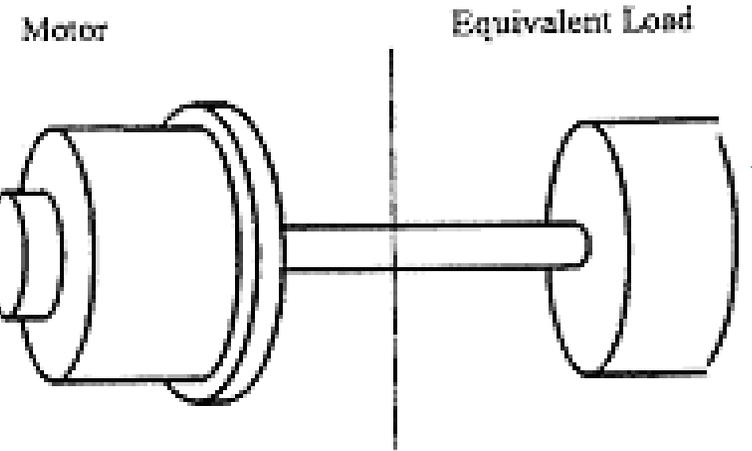
The ratio of motor inertia to reflected load inertia should be between one-to-one and up to one-to-ten.

$$\frac{J_m}{J_{eff}} = \frac{1}{1} \sim \frac{1}{10}$$

The one-to-one is considered the optimal match (an ideal case), where the motor drives a purely inertia load and this inertia ratio results in minimum heating of the motor. Let us assume that we will pick a motor which has a rotor inertia same as the load so that there is an ideal load and motor inertia match.

$$J_m = J_{eff} = 0.0012\,kg.m^2$$

$$\therefore J_{total} = J_m + J_{eff} = 0.0024\,kg.m^2$$

2) Determine the Net torque:

$$\sum T(t) = T_{total}(t) = T_m(t) - T_R(t)$$

where:

$T_{total}(t)$=the total torque.

$T_m(t)$=the torque generated by the motor.

$T_R(t)$=the resistive load torques on the system, where $T_R(t)$ represent the sum of all external torques. If the load torque is in the direction of assisting the motion, it will be negative, and net result will be the addition of two torques. The $T_R(t)$ may include friction (T_f), gravity (T_g), and process related torque and forces (i.e. an assembly application may required the mechanism to provide a desired force pressure (T_R).

$$\because T_R(t) = 0$$

$$\therefore T_{total}(t) = T_m(t)$$

3) Fundamental Equations for torque calculation:

The torque and motion relationship is:

$$J_{total}\ddot{\theta} = \sum T$$

$$(J_m + J_{eff})\ddot{\theta} = T_m$$
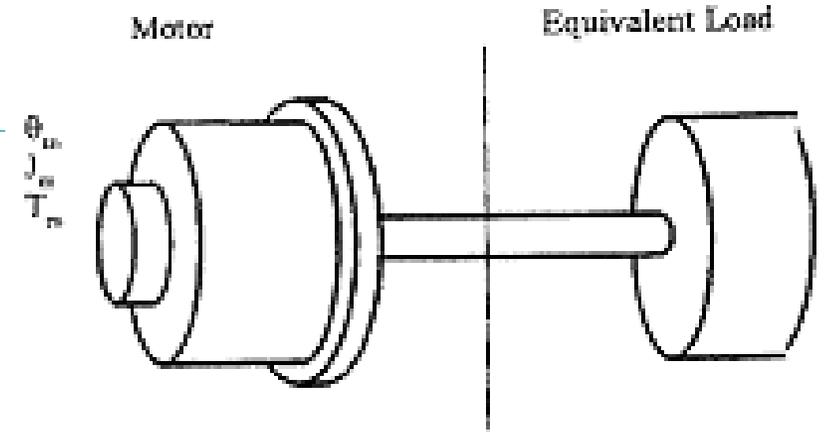
The required torque to move the load through the desired cyclic motion given in the figure can be calculated if the value of $\ddot{\theta}$ calculated.

Motor   Equivalent Load

Displacement

$\dfrac{1}{4}revolution$

θ_d

θ_r

θ_{total}

θ_a

0   ta   ta+tr   ta+tr+td   tcyc

time (sec.)
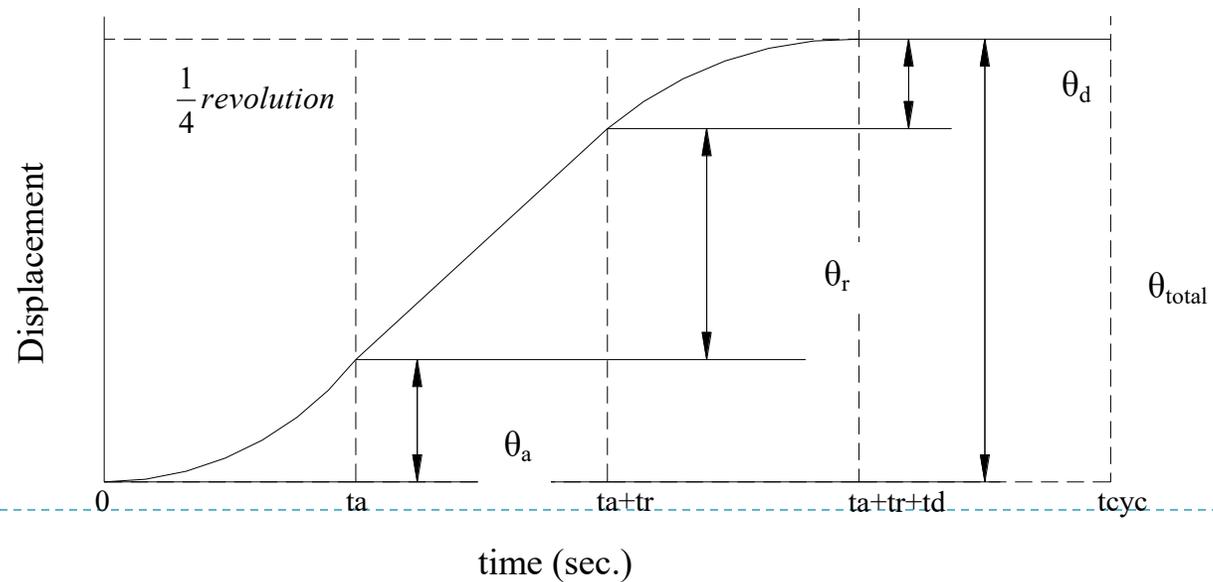
4) Define the desired cyclic motion profile in the form of load (motor) speed versus time:

From the desired motion profile specification, we can determine the velocity and acceleration of the actuator can be deliver using the kinematic relations.

$$t_a = t_r = t_d = \frac{250 - 100}{3} = 50 m \sec .$$

where:

      t_a=acceleration mode time.
      t_r=constant speed mode time.
      t_d=deceleration speed mode time.

Velocity

θ'_{max}

time (sec.)

Acceleration diagram

0

$0 \qquad ta \qquad ta+tr \qquad ta+tr+td \qquad tcyc$
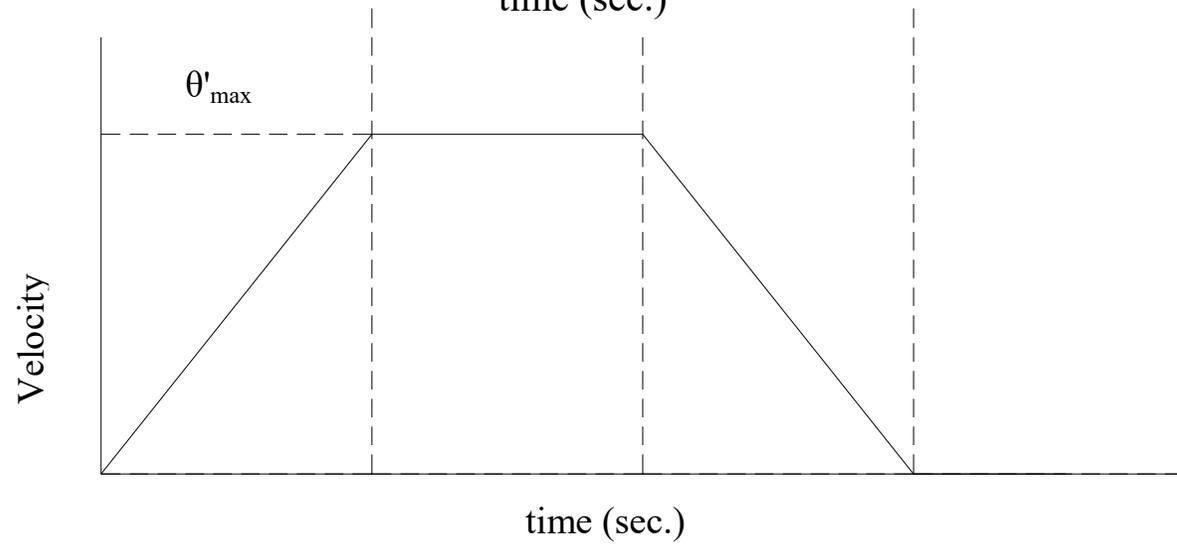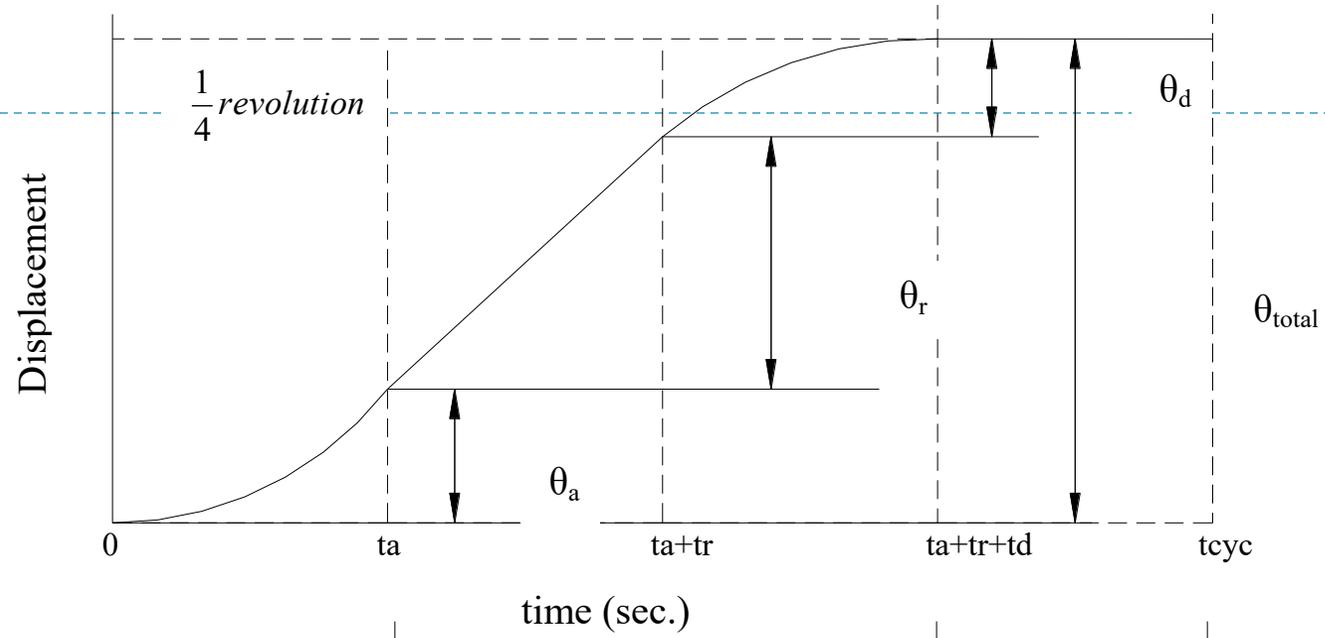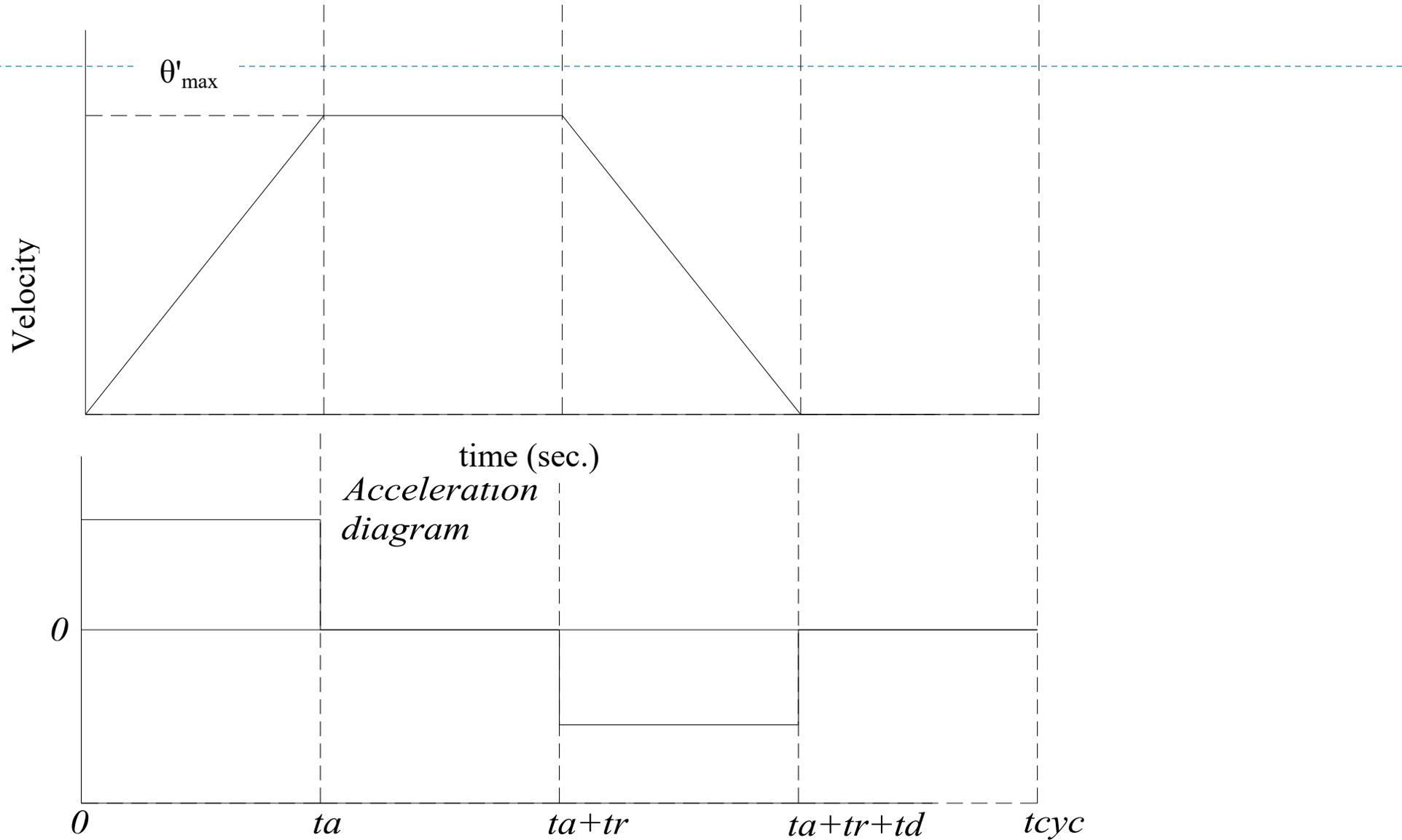
5) Calculate the maximum speed $\dot{\theta}_{max}$ required to define this profile from the motor:

From displacement motion profile:

$$\theta_{total} = \theta_a + \theta_r + \theta_d = \frac{1}{4} \times 2\pi = \frac{\pi}{2}$$

note that:

$$\theta(t) = \int \dot{\theta}(t)dt$$

From velocity motion profile:

$$\theta_{total} = \frac{1}{2}\dot{\theta}_{max}t_a + \dot{\theta}_{max}t_r + \frac{1}{2}\dot{\theta}_{max}t_d$$

$$\theta_{total} = \dot{\theta}_{max}(\frac{t_a + 2t_r + t_d}{2})$$

$$\dot{\theta}_{max} = \frac{2\theta_{total}}{t_a + 2t_r + t_d}$$

$$\therefore \dot{\theta}_{max} = \frac{\pi}{150 \times 10^{-3}} = 15.7 rad/\sec$$

$$n = \frac{60\dot{\theta}_{max}}{2\pi} = 150 rpm$$

Displacement

$\frac{1}{4}$ revolution

θ_a

θ_r

θ_d

θ_{total}

0   ta   ta+tr   ta+tr+td   tcyc

time (sec.)

Velocity

θ'_{max}

time (sec.)
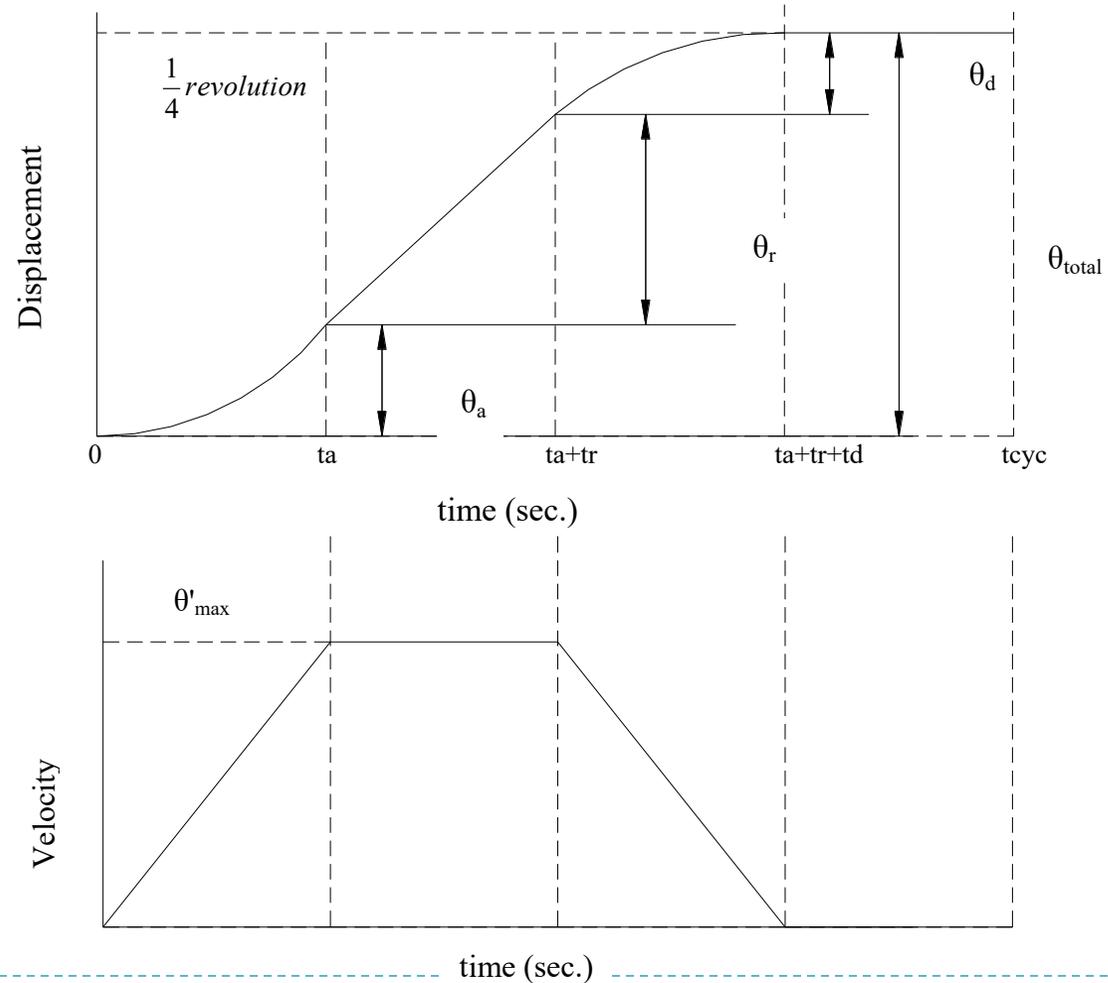
6) Calculate the angular acceleration :

Note that:

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt}$$

From velocity motion profile:

$$\ddot{\theta}_a = \frac{d\dot{\theta}_a}{dt} = \frac{\dot{\theta}_{max} - 0}{t_a - 0} = \frac{\dot{\theta}_{max}}{t_a} = \frac{15.7}{50 \times 10^{-3}} = 314 \, rad/\sec^2$$

$$\ddot{\theta}_r = \frac{d\dot{\theta}_r}{dt} = \frac{\dot{\theta}_{max} - \dot{\theta}_{max}}{(t_a + t_r) - t_a} = \frac{0}{t_r} = 0$$

$$\ddot{\theta}_d = \frac{d\dot{\theta}_d}{dt} = \frac{0 - \dot{\theta}_{max}}{(t_d + t_r + t_a) - (t_r + t_a)} = \frac{-\dot{\theta}_{max}}{t_d} = \frac{15.7}{50 \times 10^{-3}} = -314 \, rad/\sec^2$$
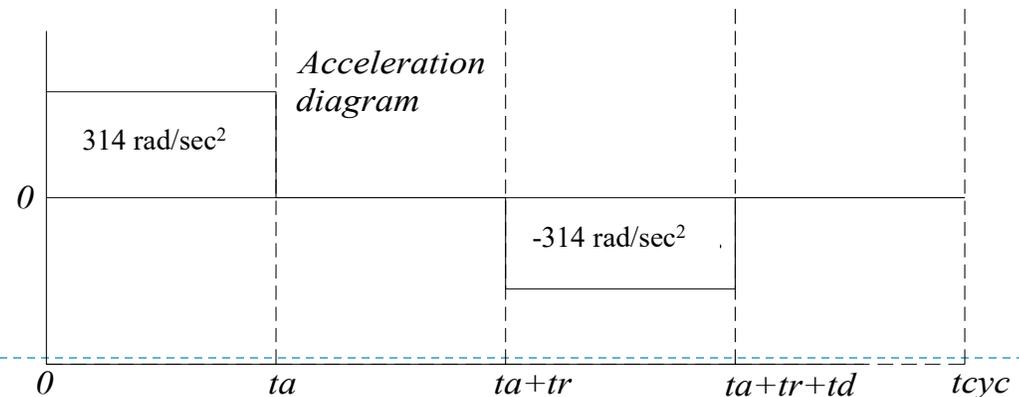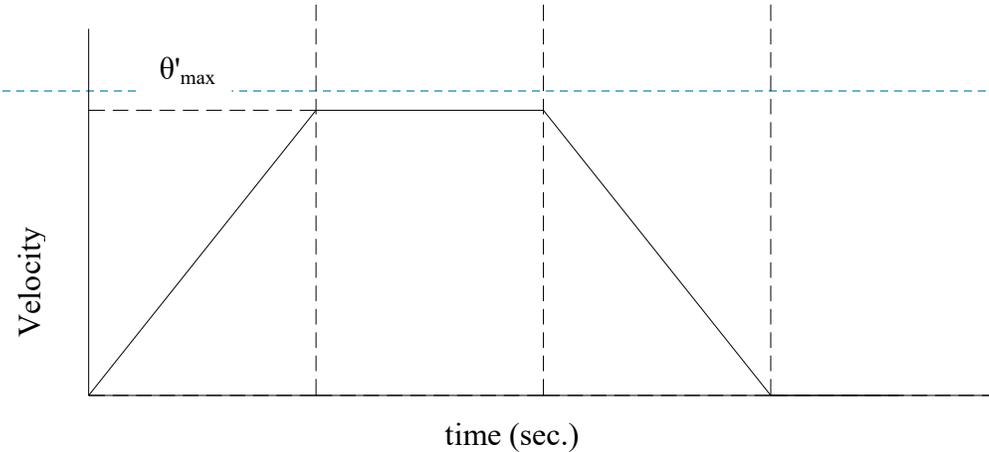
7) Use Fundamental equation for torque calculation at each mode:

$$(J_m + J_{eff})\ddot{\theta} = T_m$$

$$\therefore T_{m_a} = (J_m + J_{eff})\ddot{\theta}_a = 0.0024 \times 314 = 0.7536 N.m$$

$$\therefore T_{m_r} = (J_m + J_{eff})\ddot{\theta}_r = 0$$

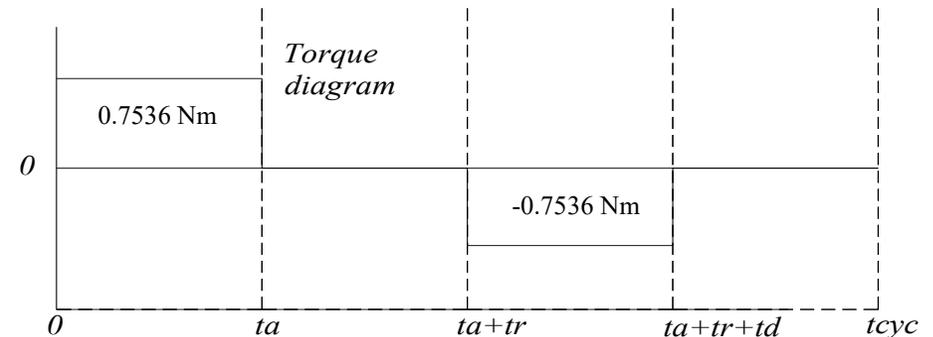$$\therefore T_{m_d} = (J_m + J_{eff})\ddot{\theta}_d = 0.0024 \times -314 = -0.7536 N.m$$
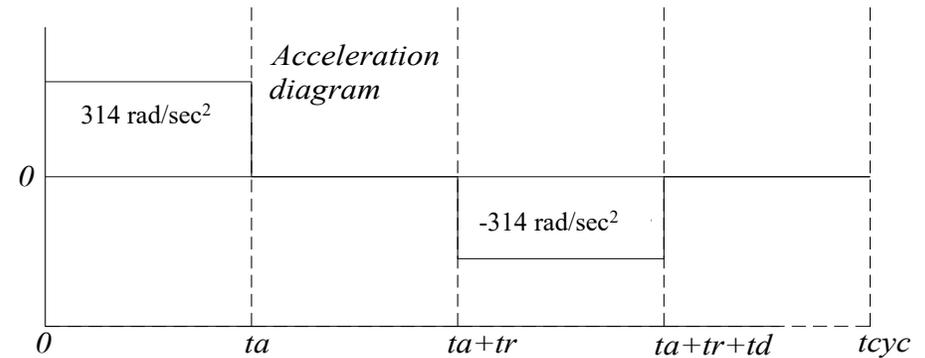
The torque diagram profile is shown in Fig

8) The Peak torque (maximum torque):
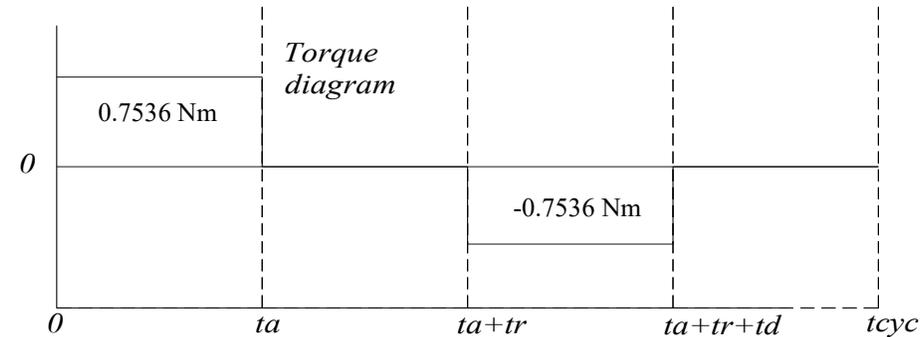
Hence, the peak torque requirement is

$$T_{max} = 0.7536 N.m$$

Acceleration diagram: 314 rad/sec², 0, -314 rad/sec², 0, ta, ta+tr, ta+tr+td, tcyc

Torque diagram: 0.7536 Nm, 0, -0.7536 Nm, 0, ta, ta+tr, ta+tr+td, tcyc

9) T_{rms}= root mean square torque over entire cycle:

$$T_{rms} = \sqrt{\frac{\int_0^{t_{cyc}} (T_m(t))^2 dt}{t_{cycle}}}$$

From torque diagram:

$$T_{rms} = \sqrt{\frac{T_{m_a}^2 t_a + T_{m_r}^2 t_r + T_{m_d}^2 t_d + T_H^2 t_{dw}}{t_a + t_r + t_d + t_{dw}}}$$

where : T_H=holding torque required in dwell mode=0

$$T_{rms} = \sqrt{\frac{(0.7536)^2 \times 50 \times 10^{-3} + 0 + (-0.7536)^2 \times 50 \times 10^{-3} + 0}{250 \times 10^{-3}}} = 0.4766 \ N.m$$

Therefore, a motor which has rotor inertia of about *0.0012 kg.m²*, maximum speed capability of *15.7 rad/sec(150 rpm)*or better, peak and RMS torque rating in the range of *0.7536 N.m* and *0.4766 N.m* range would be sufficient for the task

Torque diagram

0.7536 Nm

0

-0.7536 Nm

0   ta   ta+tr   ta+tr+td   tcyc

Mechanisms and Drives

Direct Drive

MOTOR   **LOAD**
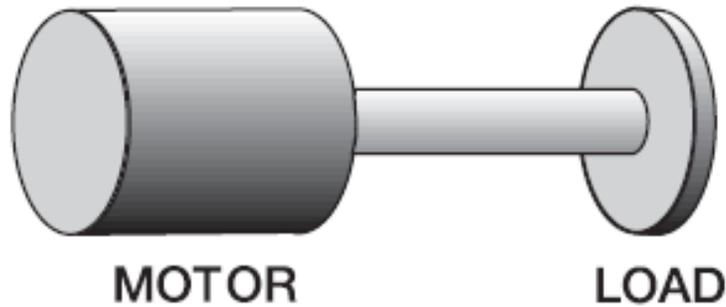
where

S_m	=	motor speed (rpm)
S_1	=	load speed (rpm)
T_m	=	motor torque (lb-in)
T_1	=	load torque (lb-in)
J_t	=	total inertia (lb-in-s^2)
J_1	=	load inertia (lb-in-s^2)
J_m	=	motor inertia (lb-in-s^2)

speed (motor) = speed (load)

$$S_m = S_1$$

torque at motor = torque at load

$$T_m = T_1$$

total inertia = inertia (load) + inertia (motor)

$$J_t = J_1 + J_m$$
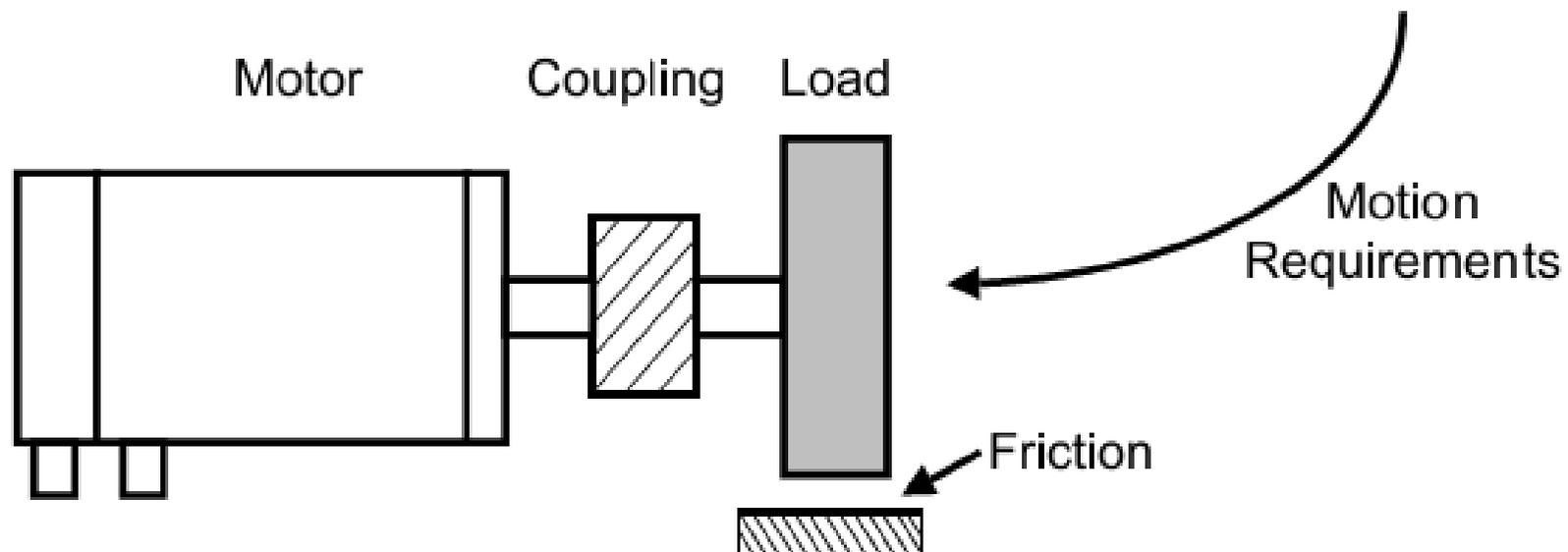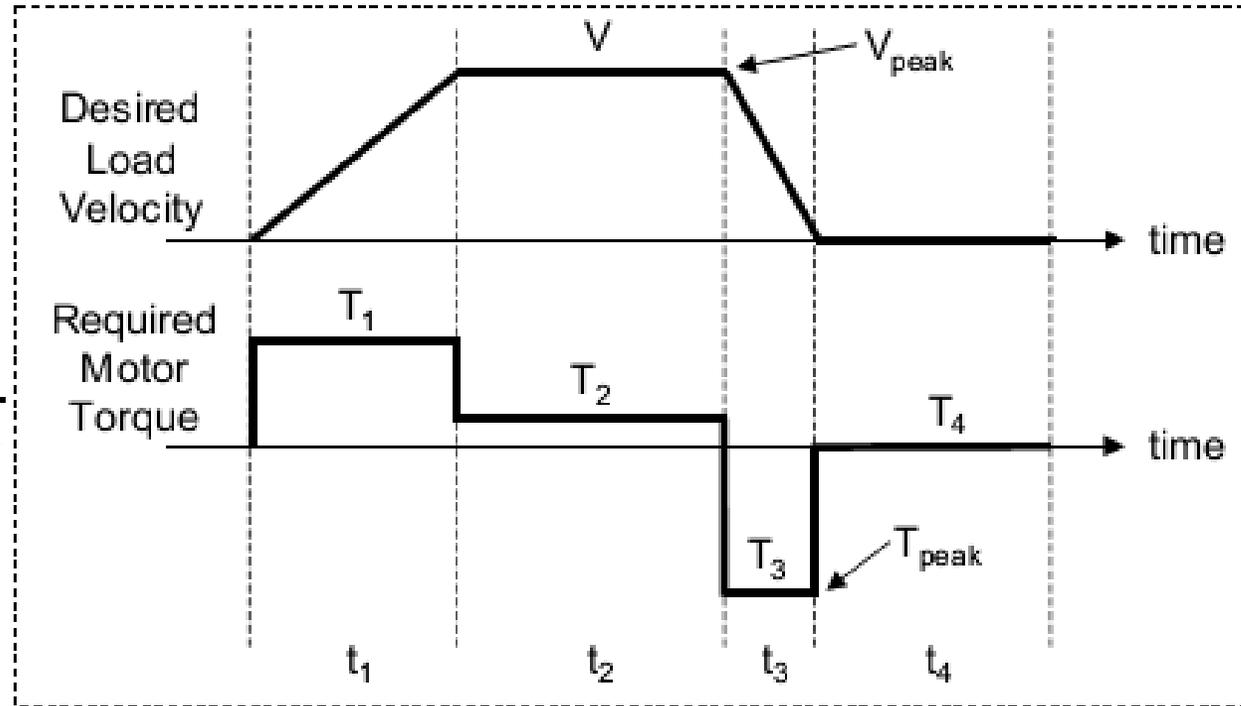
Rotating mass driven through a gear reduction.

$$T_0 = J_{eq}\alpha_1.$$

Motor — n_1

T_0

n_2 — J_2

gear ratio The speed ratio (or, the gear ratio) n is expressed by

$$n = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1} = \frac{R_1}{R_2} = \frac{n_1}{n_2}$$
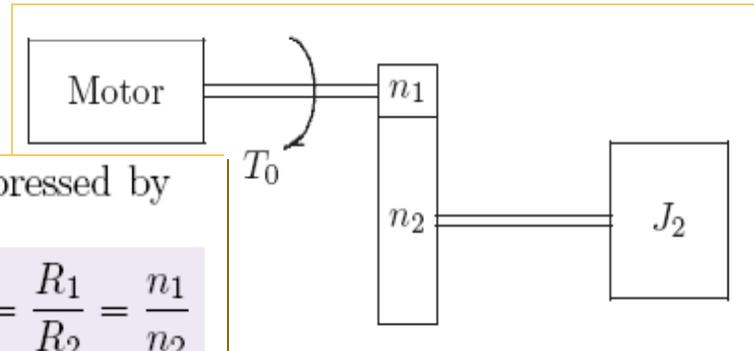
$$F = \frac{T_1}{R_1} = \frac{T_2}{R_2}$$

$$T_1 = R_1\frac{T_2}{R_2} = nT_2.$$

Taking gear 1 as a free body gives

$$T_0 - T_1 = \alpha_1 J_1$$

$$T_2 = \alpha_2 J_2.$$

The equivalent moment of inertia as

$$J_{eq} = \frac{T_0}{\alpha_1} = \frac{\alpha_1 J_1 + nT_2}{\alpha_1} = J_1 + \frac{nT_2}{\alpha_1}$$

$$= J_1 + n^2\left(\frac{T_2}{\alpha_2}\right) = J_1 + n^2 J_2.$$

How to calculate equivalent inertia?

The efficiency of a motion transmission mechanism is defined as the ratio between the output power and input power,

$$\eta = \frac{P_{out}}{P_{in}} \qquad (3.1)$$
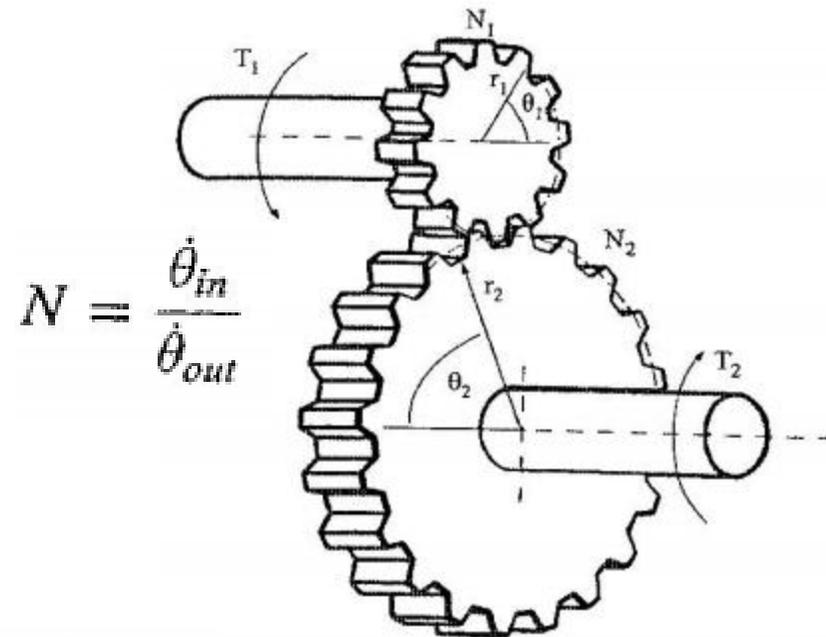
$$P_{out} = \eta \cdot P_{in}$$

$$T_{out} \cdot \dot{\theta}_{out} = \eta \cdot T_{in} \cdot \dot{\theta}_{in}$$

$$KE_l = \eta \cdot KE_{in}$$

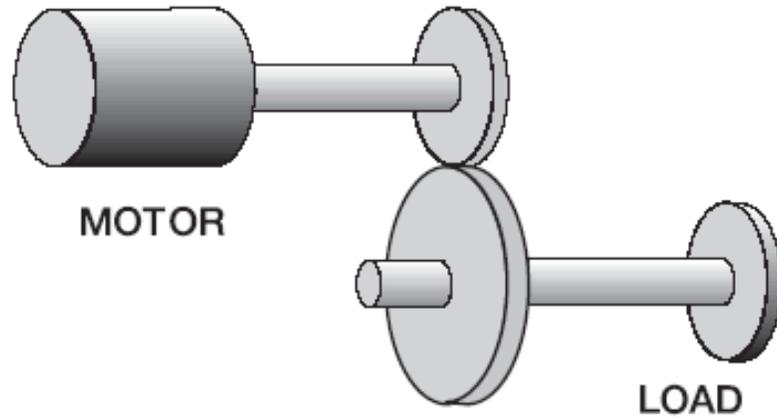$$N = \frac{\dot{\theta}_{in}}{\dot{\theta}_{out}}$$

$$KE_l = \frac{1}{2} \cdot J_l \cdot \dot{\theta}_{out}^2$$

$$= \eta \cdot \frac{1}{2} \cdot J_{in,eff} \cdot \dot{\theta}_{in}^2$$

$$= \frac{1}{2} \cdot J_l \cdot (\dot{\theta}_{in}/N)^2 \quad \Longrightarrow \quad J_{in,eff} = \frac{1}{\eta \cdot N^2} \cdot J_l$$

Gear Drive:

MOTOR

LOAD

where

S_m = motor speed (rpm)
S_1 = load speed (rpm)
N = gear ratio
N_1 = number of load gear teeth
N_m = number of motor gear teeth
T_m = motor torque (lb-in)
T_1 = load torque (lb-in)
e = efficiency
J_t = total inertia (lb-in-s²)
J_1 = load inertia (lb-in-s²)
J_m = motor inertia (lb-in-s²)

speed (motor) = speed (load) x gear ratio

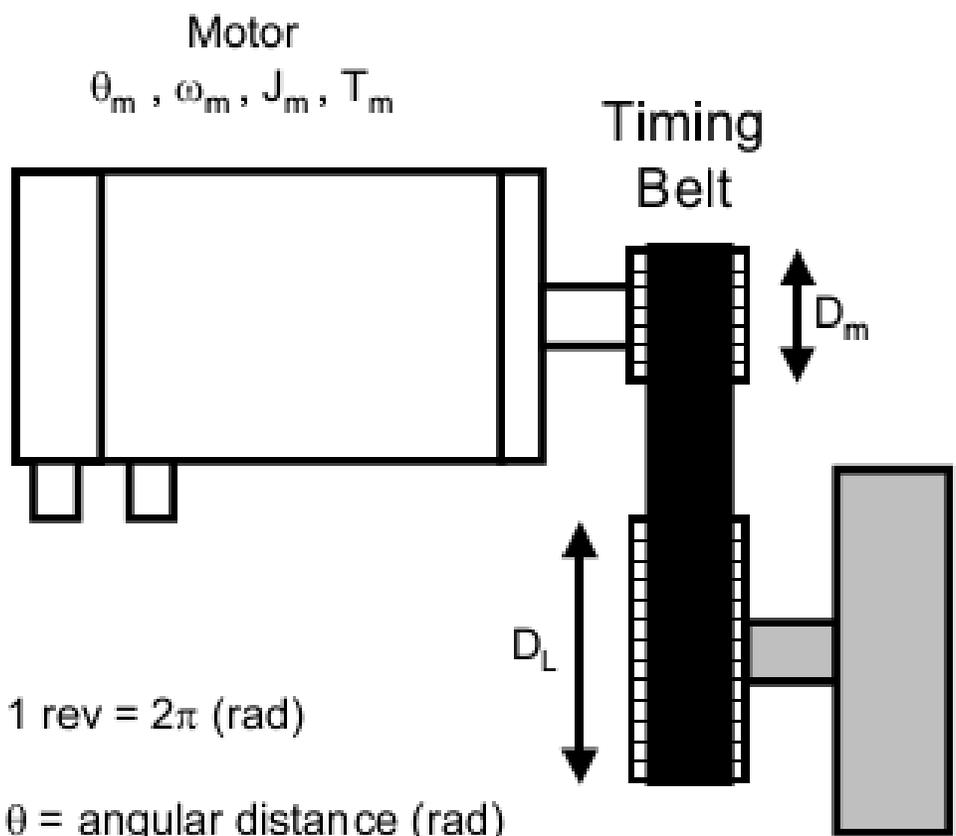$$S_m = S_1 \ x \ N$$

$$\text{or } S_m = S_1 \ x \ N_1 \ \div \ N_m$$

torque at motor = torque at load ÷ gear ratio

$$T_m = \frac{T_1}{Ne}$$

total inertia = inertia (load) ÷ (gear ratio²) + inertia (motor)

$$J_t = \frac{J_1}{N^2} + J_m$$

Motor
θ_m , ω_m , J_m , T_m

Timing Belt

D_m

D_L

Load
θ_L , ω_L , J_L , T_L

$$\text{Ratio} = N = \frac{\text{motor velocity}}{\text{load velocity}} = \frac{D_L}{D_m}$$

$$\theta_m = N\,\theta_L \ , \ \ \omega_m = N\,\omega_L$$

$$\text{Total Inertia} = J_{total} = J_m + \frac{1}{N^2}\,J_L$$

$$\text{Load Torque Reflected to Motor} = \frac{1}{N}\,T_L$$

1 rev = 2π (rad)

θ = angular distance (rad)

ω = angular velocity (rad/s)

J = moment of inertia (lb-in-s^2)

T = torque (lb-in)

Conversion of Rotary to Linear Motion

1. Rack and pinion drives,
2. Power (lead) screws,
3. Linkages.
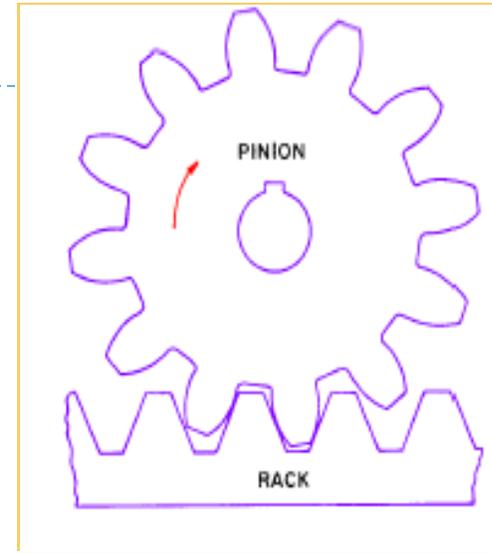
$$V = R\omega$$

If the load attached to the rack has mass *m, then,* total equivalent moment of inertia equals

$$J_{eq} = J_1 + mR^2 = J_1 + m\left(\frac{V}{\omega}\right)^2$$

Conversely, if the rack is the driver, then the moment of inertia *J1 attached* to the pinion shaft must be reflected back to the rack, and the equivalent linear inertia as felt by the pinion driving the rack is

$$m_{eq} = m + \frac{J_1}{R^2}.$$

PINION

RACK

g = gravity constant = 386 in/s^2

W_{total} = weight (lbs) = W_{load} + weight of moving parts

v = linear velocity (in/min)

x = linear distance (in)

$\theta_L = x/r$, $\omega_L = \dfrac{v}{60\,r}$, n (r.p.m.) $= \dfrac{v}{2\,\pi\,r}$

$F_{gravity} = W_{total}\,\sin\theta$, $F_{friction} = \mu\,W_{total}\,\cos\theta$

$T_L = F_{total}\,r + T_{acc}$, $T_{acc} = \left[J_{pulleys/pinion} + \dfrac{W_{total}}{g}\,r^2 \right] \dfrac{dn}{dt}$

$F_{total} = F_{external} + F_{friction} + F_{gravity}$

← $F_{external}$

← $F_{gravity}$

→ $F_{friction}$

Load (W_{load})

r = drive radius (in)

Incline angle θ

N:1

Motor
θ_m , ω_m , J_m , T_m

Optional Timing Belt or Gear Reducer

Load Side
θ_L , ω_L , J_L , T_L

$F_{external}$ $F_{total} = F_{external} + F_{friction} + F_{gravity}$

$F_{gravity}$

Load Side
θ_L , ω_L , J_L , T_L $F_{friction}$

Load (W_{load})

Table (W_{table})

μ = coefficient of friction)

Motor
θ_m , ω_m , J_m , T_m

N:1

Incline angle

θ

Optional Timing Belt or Gear Reducer

Ball Nut   Ball Screw (J_{screw})
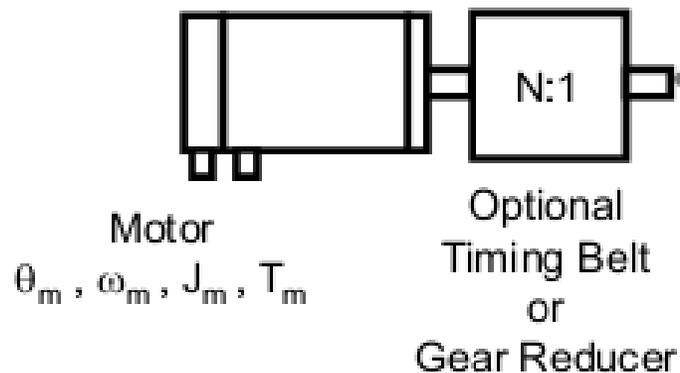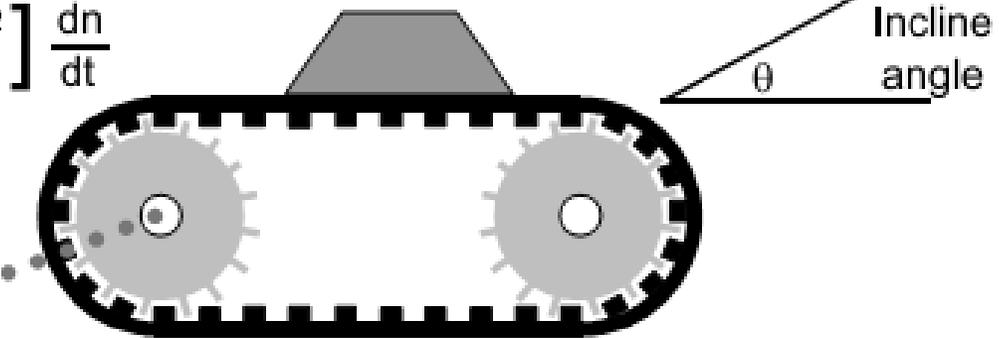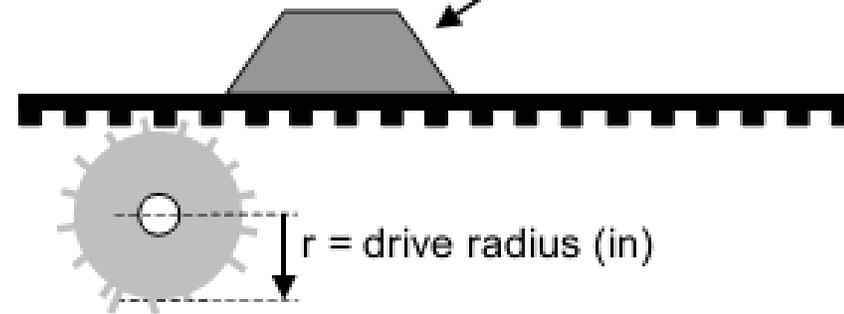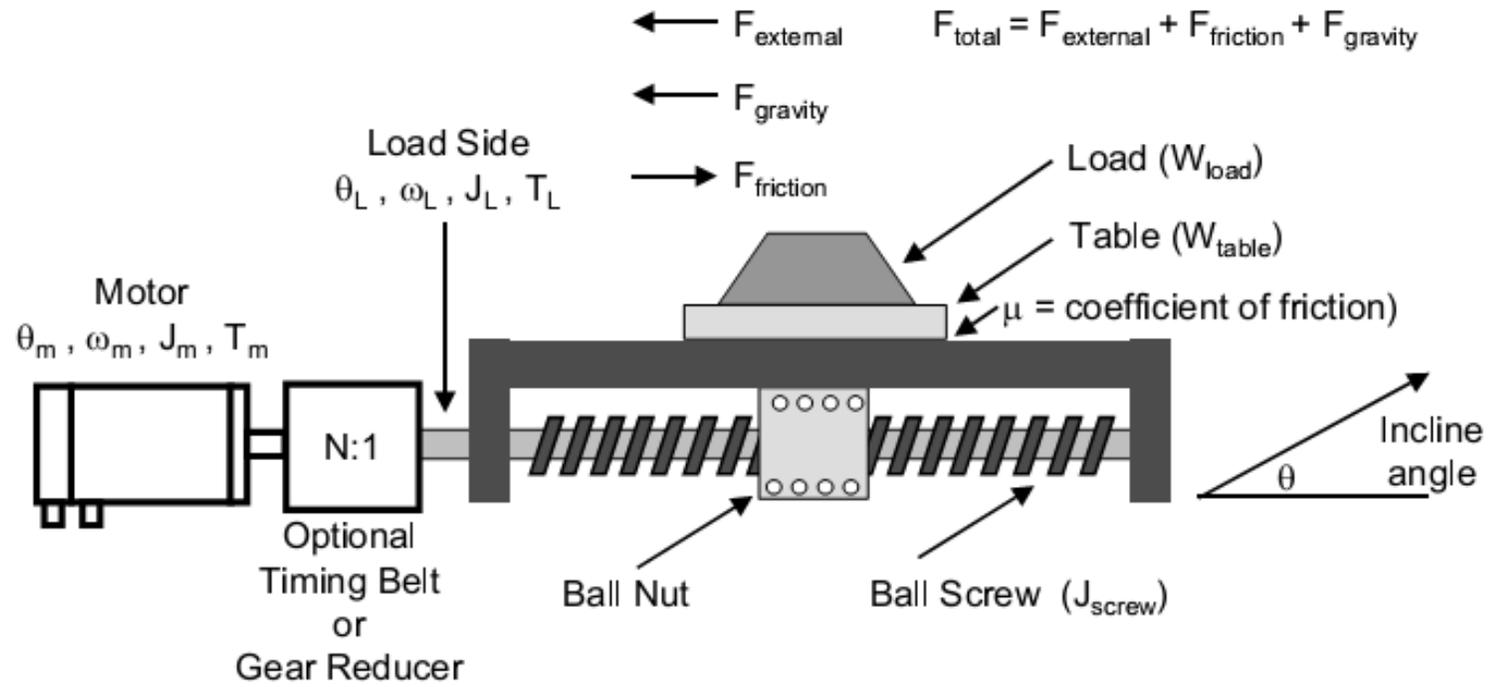
P = pitch = revs/inch of leadscrew

h = lead = inches/rev of leadscrew

g = gravity constant = 386 in/s^2

W_{total} = weight (lbs) = $W_{load} + W_{table}$

v = linear velocity (in/min)

x = linear distance (in)

$$\theta_L = 2\pi P x \ , \ \omega_L = \frac{2\pi P v}{60} \ , \ n \ (r.p.m.) = P v$$

$$F_{gravity} = W_{total} \sin\theta \ , \ F_{friction} = \mu W_{total} \cos\theta$$

$$T_L = \frac{F_{total}}{2\pi P} + T_{acc} \ , \ T_{acc} = \left[J_{screw} + \frac{W_{total}(1/2\pi P)^2}{g} \right] \frac{dn}{dt}$$
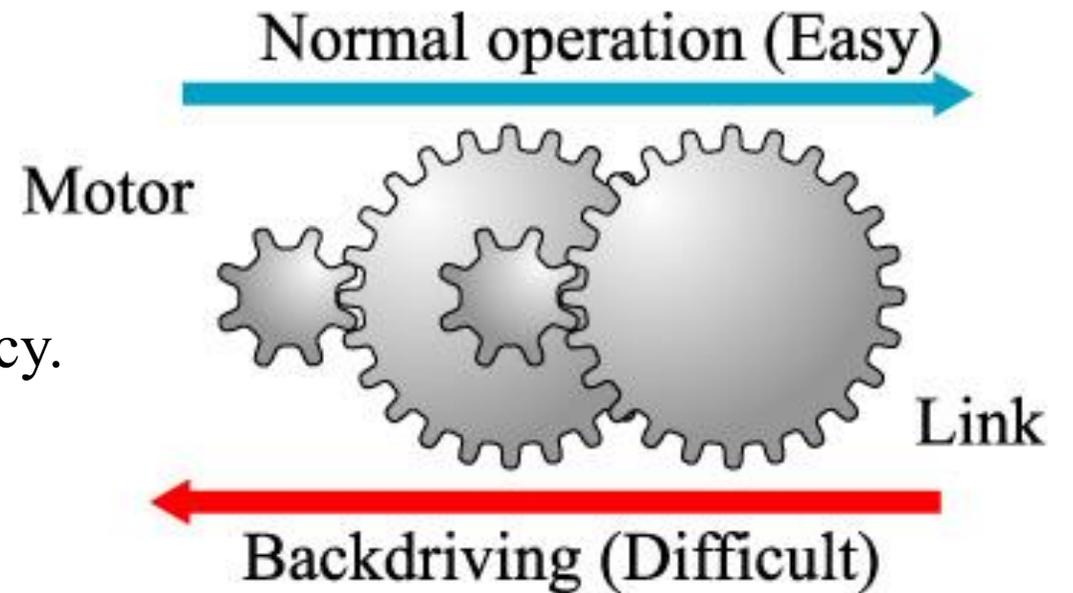
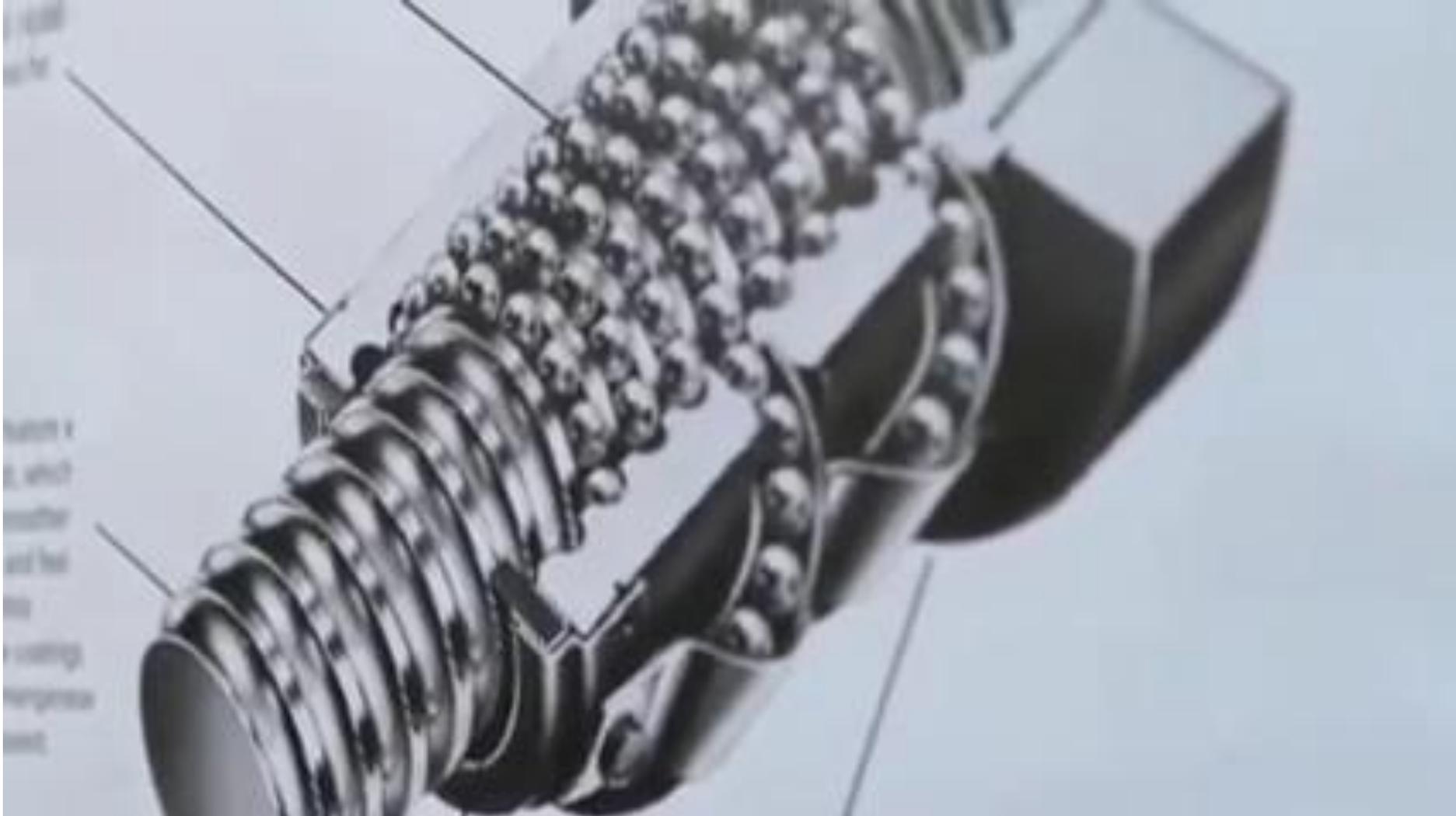# Backdrivability (Passive behavior)

$$\eta_{back-drive} = 2 - \frac{1}{\eta_{forward}}$$

Where:
$\eta{back\text{-}drive}$, back-driving or reverse efficiency.
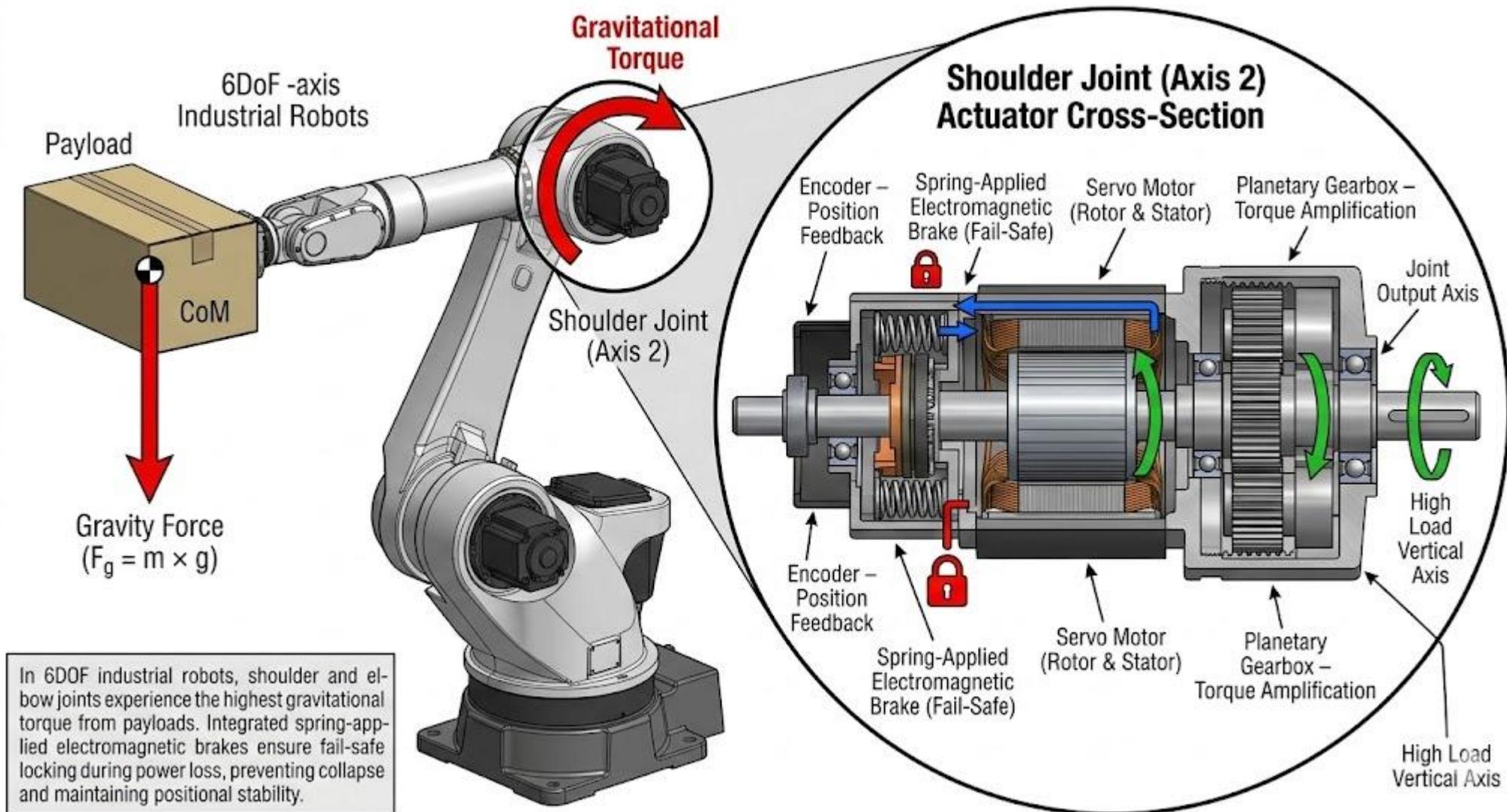$\eta{forward}$, driving or forward efficiency.

Normal operation (Easy)

Motor

Link

Backdriving (Difficult)

Backdrivability (Passive behavior)

Robotic system actuation system

6DoF -axis Industrial Robots

Payload
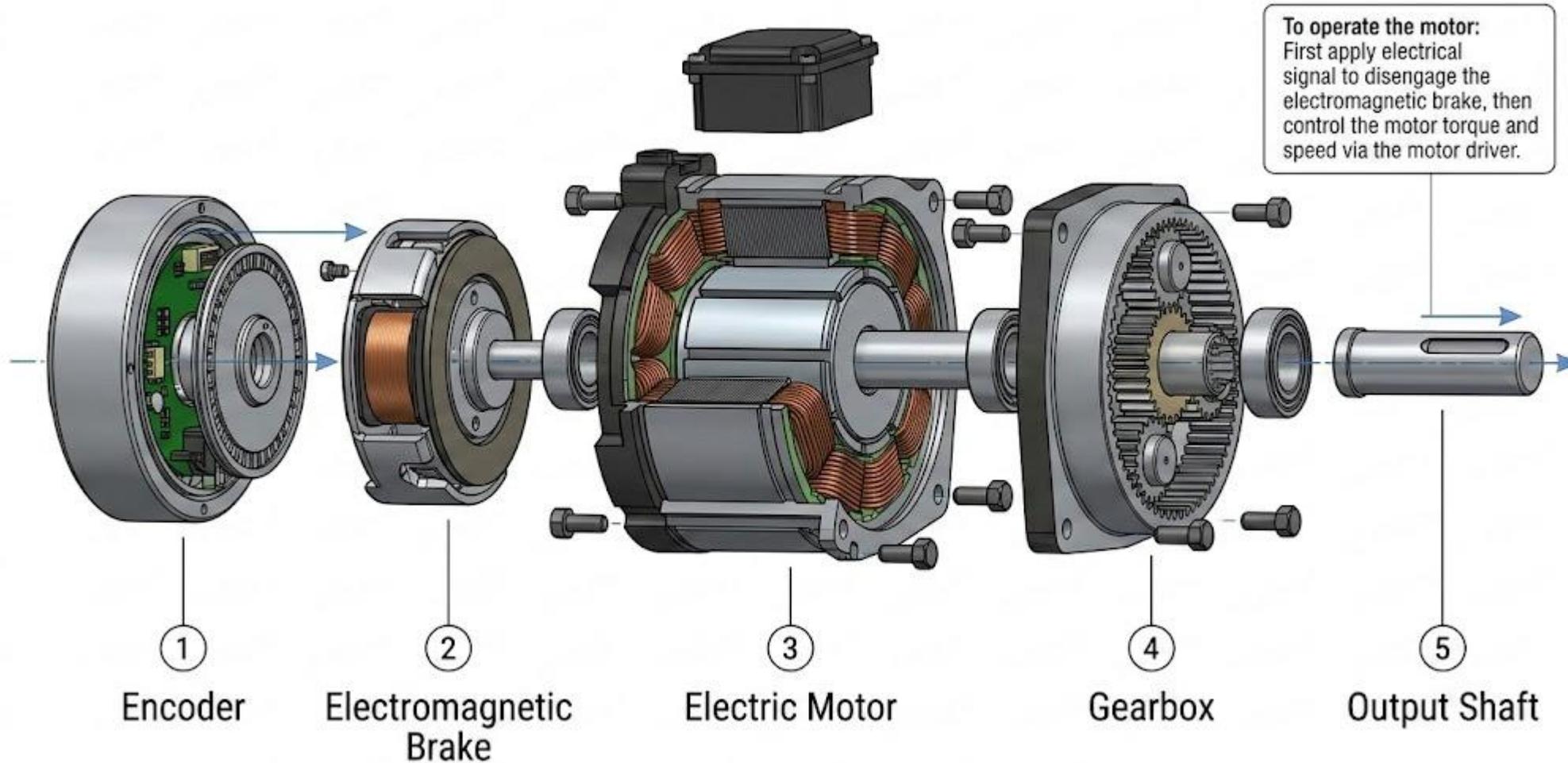
CoM

Gravity Force
$(F_g = m \times g)$

Gravitational Torque
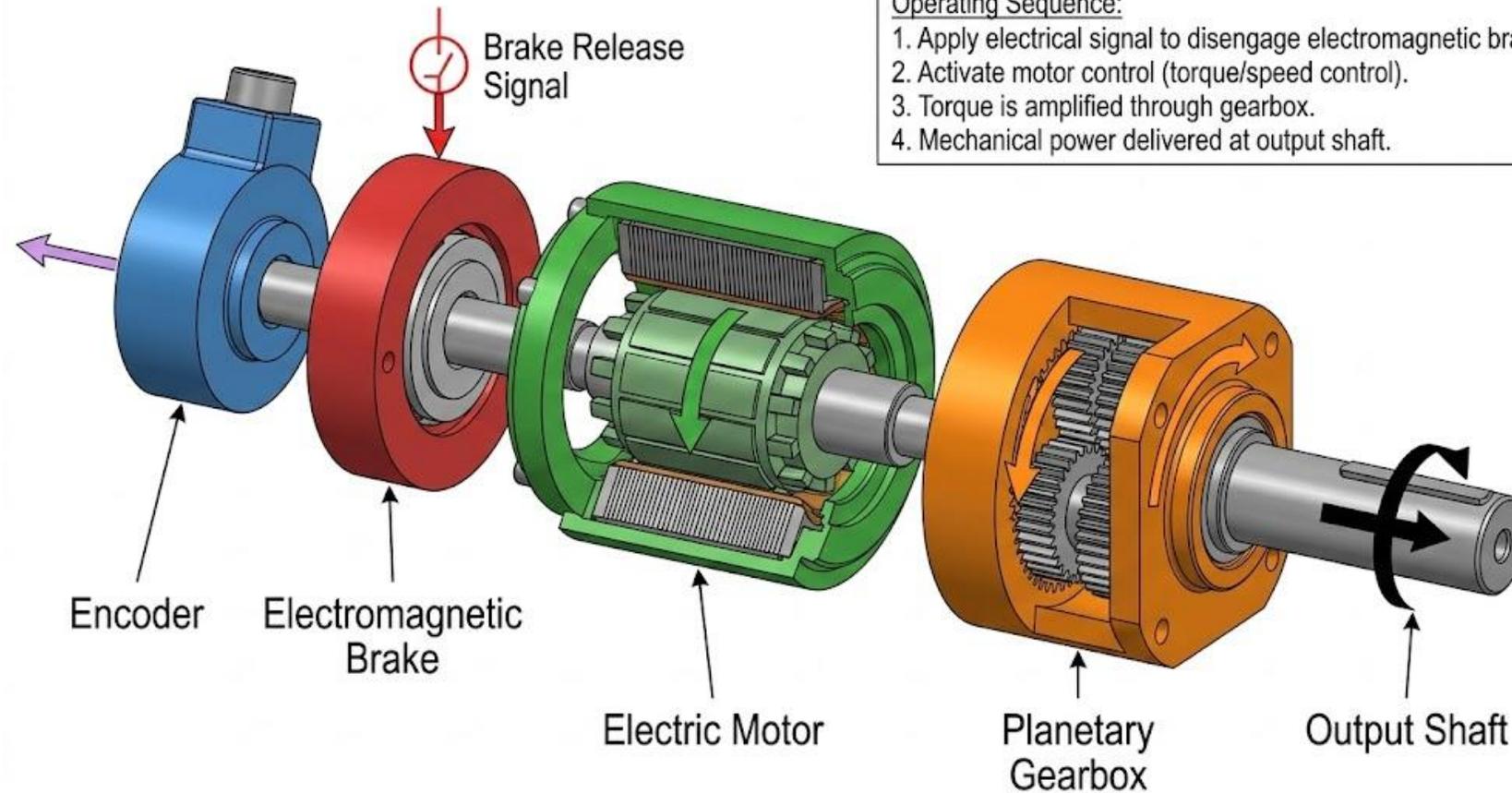
Shoulder Joint (Axis 2)

In 6DOF industrial robots, shoulder and elbow joints experience the highest gravitational torque from payloads. Integrated spring-applied electromagnetic brakes ensure fail-safe locking during power loss, preventing collapse and maintaining positional stability.

Shoulder Joint (Axis 2) Actuator Cross-Section

Encoder – Position Feedback

Spring-Applied Electromagnetic Brake (Fail-Safe)

Servo Motor (Rotor & Stator)

Planetary Gearbox – Torque Amplification

Joint Output Axis

Encoder – Position Feedback

Spring-Applied Electromagnetic Brake (Fail-Safe)

Servo Motor (Rotor & Stator)

Planetary Gearbox – Torque Amplification

High Load Vertical Axis

High Load Vertical Axis

Servomotor with Electromagnetic Brake

To operate the motor:
First apply electrical signal to disengage the electromagnetic brake, then control the motor torque and speed via the motor driver.

① Encoder   ② Electromagnetic Brake   ③ Electric Motor   ④ Gearbox   ⑤ Output Shaft

Servomotor with Electromagnetic Brake

Brake Release Signal

Operating Sequence:
1. Apply electrical signal to disengage electromagnetic brake.
2. Activate motor control (torque/speed control).
3. Torque is amplified through gearbox.
4. Mechanical power delivered at output shaft.

Encoder

Electromagnetic Brake

Electric Motor

Planetary Gearbox

Output Shaft

Servomotor with Electromagnetic Brake

Servomotor Internal Structure and Operating Sequence

Controller

Encoder (Position Feedback) → Electromagnetic Brake → Electric Motor (Torque Generation) → Gearbox (Torque Amplification) → Output Shaft (Mechanical Output)
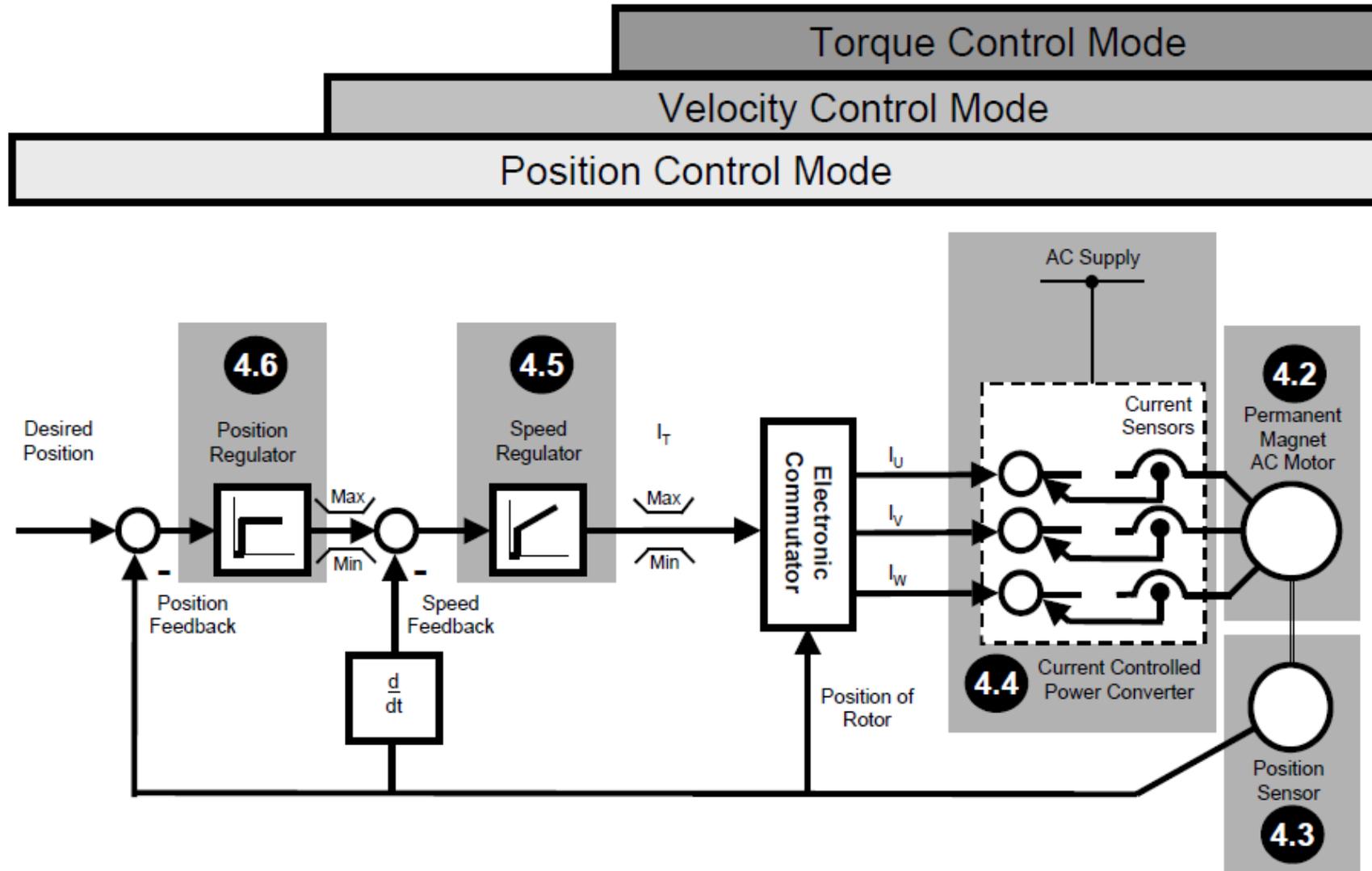
Brake Release Signal

Motor Control Signal
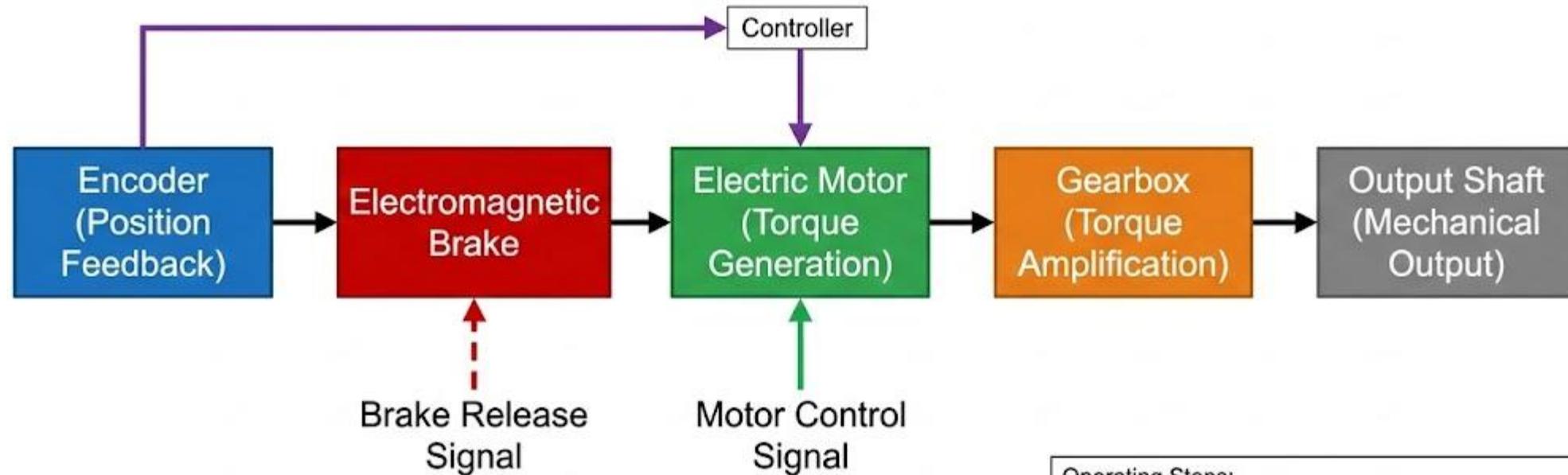
Operating Steps:
1. Apply electrical signal to disengage the brake.
2. Send control signal to motor.
3. Torque passes through gearbox.
4. Output motion delivered at shaft.

Cascade Control Structure of High Performance DC Servo System
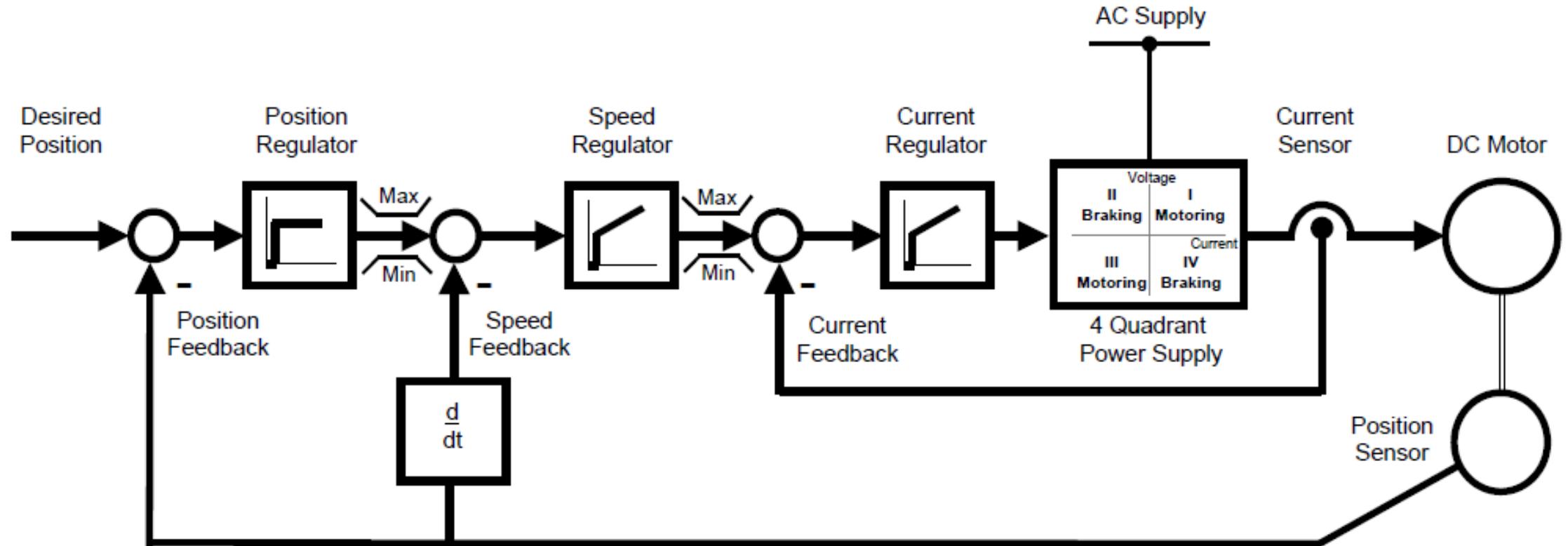
Block Diagram of AC Servo System

Questions

Thank You