MCT344 & MCT342

Industrial Robotics

Sheet (2): Part#1

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Sheet (2) – Pb.1

Consider the following sequence of rotations:

1. Rotate by $\boldsymbol{\phi}$ about the world **x-axis**.
2. Rotate by $\boldsymbol{\theta}$ about the current **z-axis**.
3. Rotate by $\boldsymbol{\psi}$ about the world **y-axis**.

Write the matrix product that will give the resulting rotation matrix **(do not perform the matrix multiplication).**

Solution:

World/Fixed: Pre-Multiplication.
Current: Post-Multiplication.

$$\text{R} = R_{y,\boldsymbol{\psi}} R_{x,\boldsymbol{\phi}} R_{z,\boldsymbol{\theta}}$$

Sheet (2) – Pb.2

Consider the following sequence of rotations:

1. Rotate by $\boldsymbol{\phi}$ about the world **x-axis**.
2. Rotate by $\boldsymbol{\theta}$ about the current **z-axis**.
3. Rotate by $\boldsymbol{\psi}$ about the current **x-axis**.
4. Rotate by $\boldsymbol{\alpha}$ about the world **z-axis**.

Write the matrix product that will give the resulting rotation matrix **(do not perform the matrix multiplication).**

Solution:

World/Fixed: Pre-Multiplication.
Current: Post-Multiplication.

$$\mathrm{R} = R_{z,\alpha} R_{x,\boldsymbol{\phi}} R_{z,\theta} R_{x,\boldsymbol{\psi}}$$

Sheet (2) – Pb.3

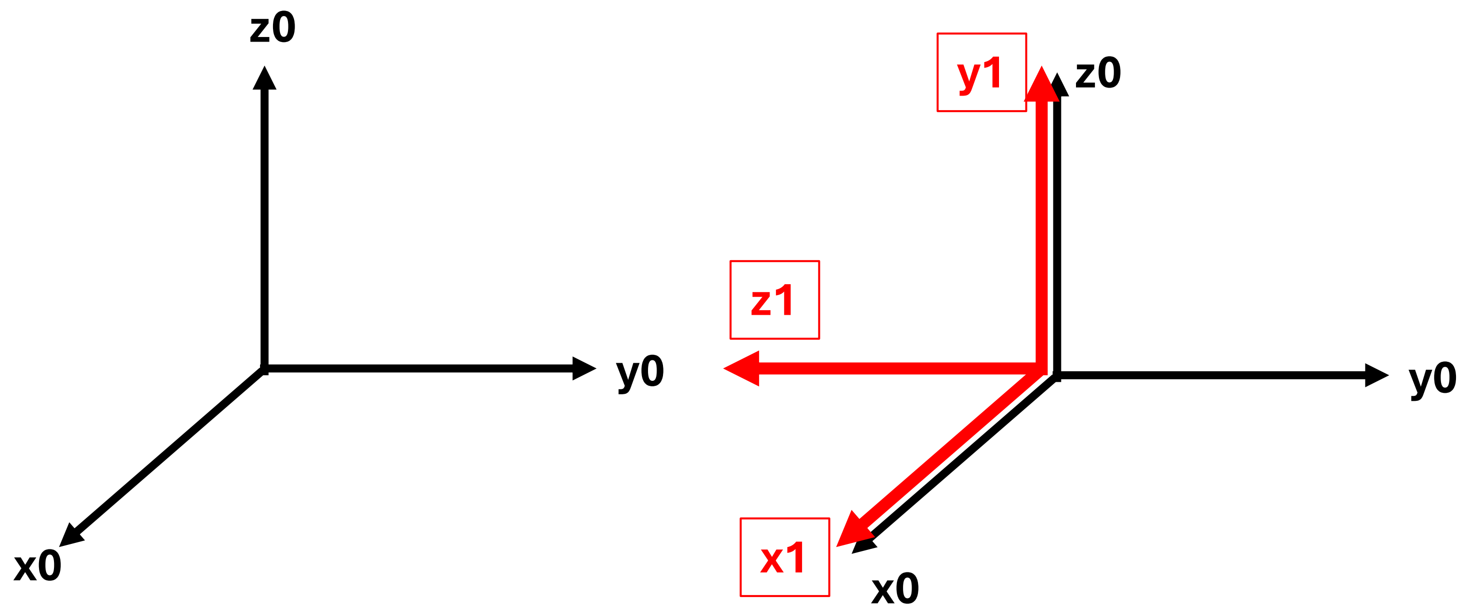
If the coordinate frame $\mathbf{o_1x_1y_1z_1}$ is obtained from the coordinate frame $\mathbf{o_0x_0y_0z_0}$ by a rotation of $\frac{\pi}{2}$ about the **x-axis** followed by a rotation of $\frac{\pi}{2}$ about the fixed **y-axis**, find the rotation matrix ***R*** representing the composite transformation. Sketch the initial and final frames.

Solution:

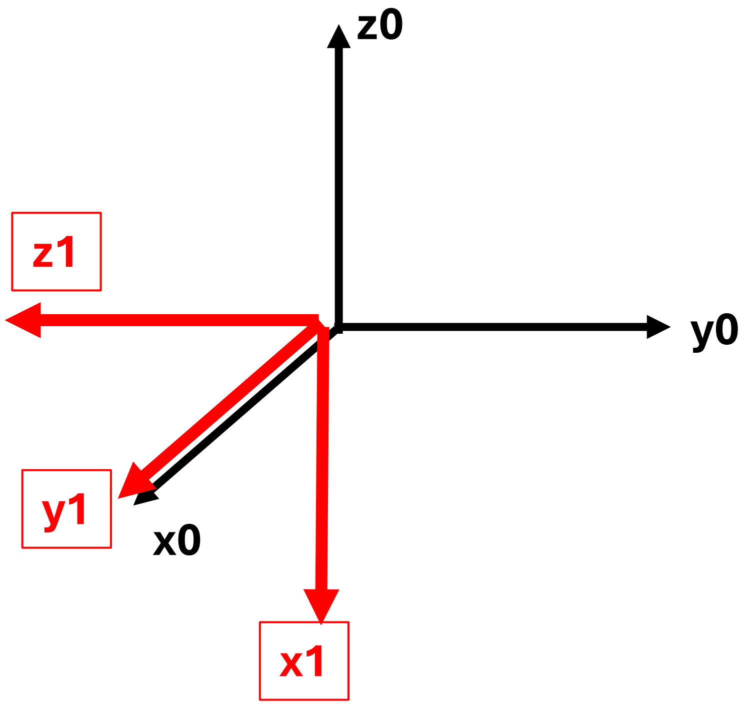
$$R = R_{y,\frac{\pi}{2}} R_{x,\frac{\pi}{2}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

Solution:



rotation of $\frac{\pi}{2}$ about the x-axis



$$\begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

rotation of $\frac{\pi}{2}$ about the fixed y-axis

Sheet (2) – Pb.4

Suppose that three coordinate frames $\mathbf{o_1x_1y_1z_1}$, $\mathbf{o_2x_2y_2z_2}$, and $\mathbf{o_3x_3y_3z_3}$ are given, and suppose:

$$\boldsymbol{R_2^1} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\mathbf{1}}{\mathbf{2}} & -\frac{\sqrt{\mathbf{3}}}{\mathbf{2}} \\ \mathbf{0} & \frac{\sqrt{\mathbf{3}}}{\mathbf{2}} & \frac{\mathbf{1}}{\mathbf{2}} \end{bmatrix}, \boldsymbol{R_3^1} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Find the matrix $\boldsymbol{R_3^2}$

Solution:

$$\boldsymbol{R_3^1} = \boldsymbol{R_2^1 R_3^2} \rightarrow \boldsymbol{R_3^2} = \boldsymbol{R_2^1}^{-1}\,\boldsymbol{R_3^1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}$$

Sheet (2) – Pb.6

Compute the homogeneous transformation representing a translation of 3 units along the **x-axis** followed by a rotation of $\frac{\pi}{2}$ about the current **z-axis** followed by a translation of 1 unit along the fixed **y-axis**. Sketch the frame. What are the coordinates of the origin O_1 with respect to the original frame in each case?

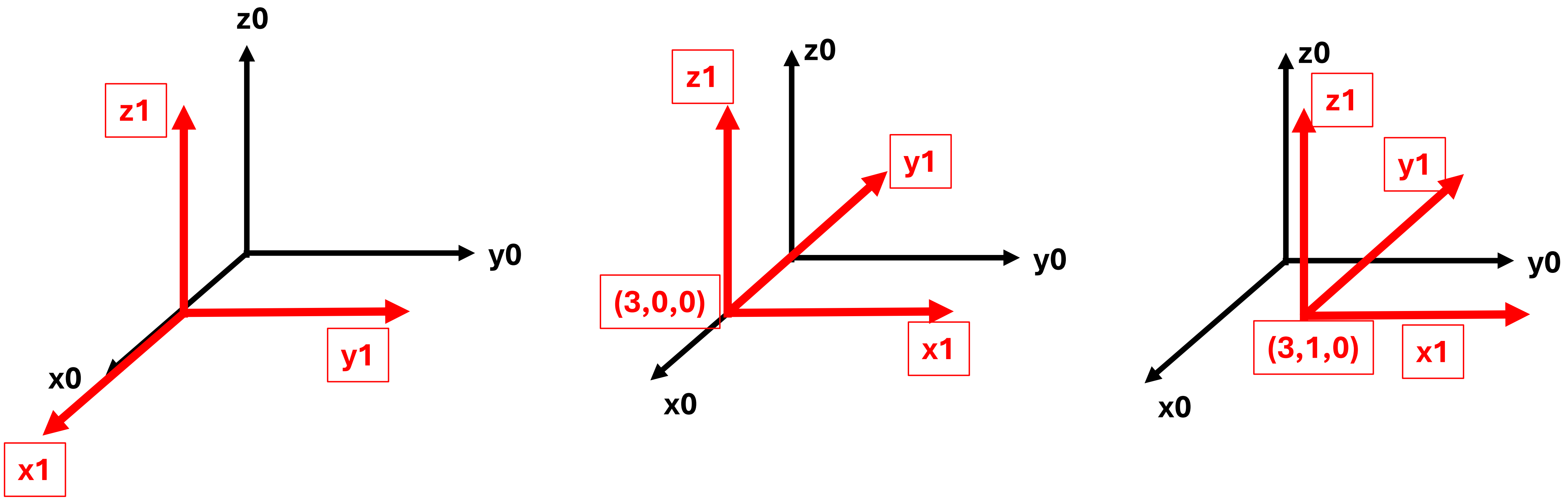
Solution:

Trans $= d_{y,1} d_{x,3} R_{z,\frac{\pi}{2}}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution:



Origin O_1 lies in (3,1,0)

Any questions?