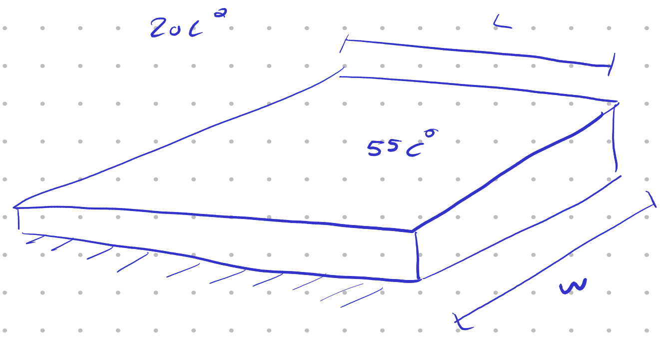
4. It is required to design a computer processor, the processor may be considered as a hot plate insulated from one of its surfaces. The temperature of the other surface is 55 °C. It is required to cool the processor by natural convection using ambient air at 20 °C. It is suggested to make the surface of the processor either rectangular with dimensions 5.5 cm × 1.5 cm or having a circular shape with the same surface area. Calculate the heat transfer from both shapes and find out whether it is better to make the surface rectangular or circular. *Consider the hot surface of the processor is facing up.*



$$T_{film} = \frac{T_s + T_\infty}{2} = \frac{55+20}{2} = 37.5\ c°$$

$$\beta = \frac{1}{37.5 + 273.15} = 3.22 \times 10^{-3}\ \frac{1}{K}$$

Properties of air at 1 atm pressure

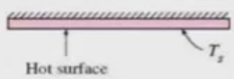
Temp. T, °C	Density ρ, kg/m³	Specific Heat c_p, J/kg·K	Thermal Conductivity k, W/m·K	Thermal Diffusivity α, m²/s	Dynamic Viscosity μ, kg/m·s	Kinematic Viscosity ν, m²/s	Prandtl Number Pr
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255

$k = 0.026435\ W/m{\cdot}K$ $\qquad \nu = 1.6785 \times 10^{-5}$

$Pr = 0.72615$

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu	
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11}	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$	(9–22) (9–23)
(b) Lower surface of a hot plate (or upper surface of a cold plate) 		10^5–10^{11}	$Nu = 0.27Ra_L^{1/4}$	(9–24)

$$L_c = A_s/p = \frac{LW}{2L+2W} = \frac{5.5 \times 1.5}{2 \times (5.5+1.5)} = 0.59\ cm$$

$$Ra_{L_c} = Gr\,Pr = \frac{(9.81)\,\beta\,(T_s - T_\infty)\,L_c^3}{\nu^2}\,Pr = 551.796$$

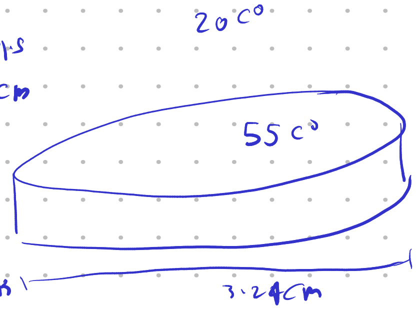
$$Nu_{L_c} = 0.54\,Ra_{L_c}^{1/4} = 2.617 = \frac{h_1 L_c}{k} \rightarrow h_1 = 11.72\ W/m^2K$$

$$\dot{Q}_1 = h_1 A (T_s - T_\infty) = 0.34\ W$$

$$L_c = \frac{5.5 \times 1.5}{\pi (3.24)} = 0.81\,cm$$

$A_s = \pi/4\, d^2 = 5.5 \times 1.5$

$\rightarrow d = 3.24\,cm$



$Ra_{L_c} = 1514.37$

$$Nu_{L_c} = 0.59\, Ra_{L_c}^{1/4} = 3.37 = \frac{h_2 L_c}{k} \longrightarrow h_2 = 10.99\, W/m^2K$$

$$\dot{Q}_2 = h_2 A (T_s - T_\infty) = 0.32\, W$$

$$\dot{Q}_1 > \dot{Q}_2 \longrightarrow \text{rectangle}$$

7. A 12 cm wide and 18 cm high vertical hot surface in a 30 °C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction and have height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80 °C. All properties are evaluated at film temperature. For optimum configuration determine the fin efficiency, effectiveness and the overall efficiency.

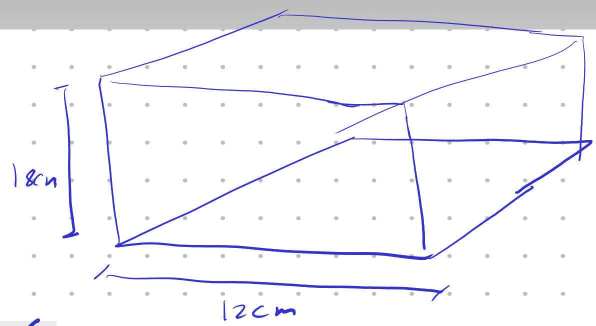
$$S_{opt} = 2.714 \frac{L}{Ra_L^{0.25}}$$

$$Nu_{opt} = 1.307$$

$t = 0.1\,cm \qquad = 18\,cm$

$$T_{film} = \frac{T_s + T_\infty}{2} = 5.5\, C°$$

$$\beta = \frac{1}{5.5 + 273.15} = 3.589\ 1/K$$



Properties of air at 1 atm pressure

Temp. T, °C	Density ρ, kg/m³	Specific Heat c_p, J/kg·K	Thermal Conductivity k, W/m·K	Thermal Diffusivity α, m²/s	Dynamic Viscosity μ, kg/m·s	Kinematic Viscosity ν, m²/s	Prandtl Number Pr
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202

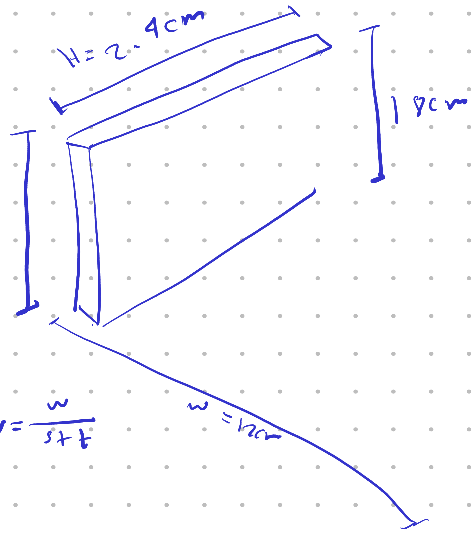
$L = L_c$

$Ra = ✓$

$S_{opt} = ✓$

$$Nu_{opt} = \frac{S_{opt}\, h}{k} \rightarrow h = ✓$$

* vertical plate $L = L_c$



$N = \frac{w}{s+t}$

$A = 2LHN$

$$Q = A\, h\, (T_s - T_\infty)$$

12. In order that a PCB (printed circuit board) used in the control of a gas turbine to operate properly its surface temperature must not exceed 40°C when placed horizontally facing up in ambient air of 25°C. The dimensions of the PCB is 30×20 cms. Calculate the amount of heat dissipated in this case. If the same PCB is to be used in desert place where it has to be placed inside a horizontal enclosure of dimensions 30×20×5 cms. If the amount of heat to be dissipated by the PCB is as calculated and its surface temperature must not exceed 40°C (consider this temperature as the hot surface of the PCB), find the temperature of the cold surface of the PCB to maintain safe operating conditions.

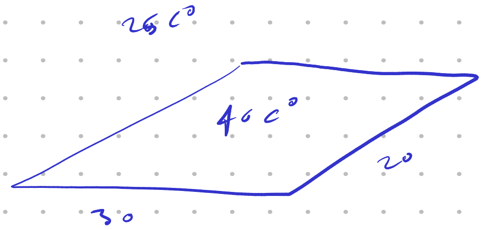
$T_f = ✓$

→ $\beta = ✓$ → $k = ✓$, $Pr = ✓$, $\nu = ✓$

→ $L_c = A/p$ ✓

→ $Ra = ✓$

→ Nu → h ✓ → $\dot{Q}_1 = ✓$



$\dot{Q}_1 = \dot{Q}_2$

$T_{film} = \frac{T_H + T_c}{2}$ → assume

→ β → 13, Pr, $\nu = ✓$

→ L_c

→ Ra → Nu → h → T_c

$\dot{Q}$