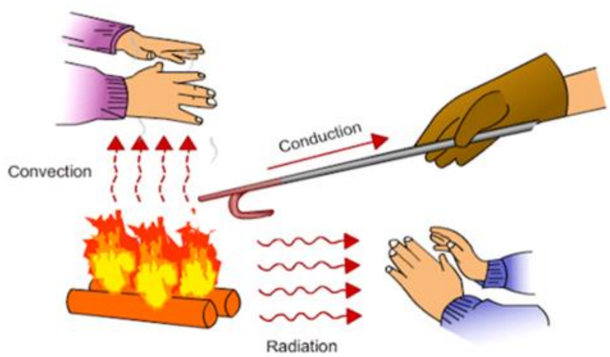
One dimension steady state conduction heat transfer

Heat transfer describes the flow of heat (thermal energy) due to temperature difference.

Modes of heat transfer

ايه هى الطرق اللى بيتنقل بيها الحراره ؟

1. conduction.
2. Convection.
3. Radiation.



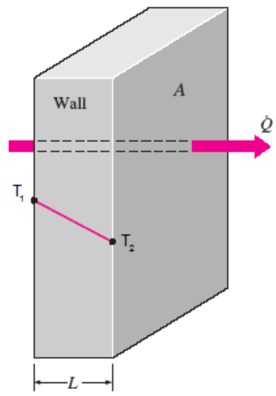
Conduction

Is the process by which heat energy is transmitted from more energetic to less energetic particles.

In solid, it is due to vaibration of molecules and energy transport by free electrons.

In gases and liquids, conduction is due to collisions and diffusion of molecules.

Fourier's law of heat conduction



$$\text{Rate of heat transfer} \propto \frac{\overbrace{A}^{\text{Area}} * \overbrace{dT}^{\text{Temp. difference}}}{\underbrace{dx}_{\text{thickness}}}$$

$$\therefore \quad Q_{cond} = -KA\frac{dT}{dx} = -KA\frac{\Delta T}{\Delta x} \quad [W]$$

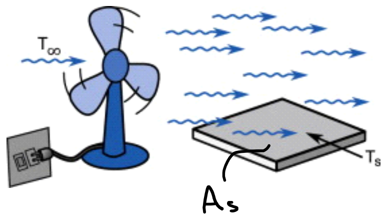
Thermal conductivity "K" $[W/mK]$ it is the material's ability to conduct heat.

$K_{diamond} = 2300\ W/mK$, $K_{iron} = 80.2\ W/mK$, $K_{wood} = 0.17\ W/mK$

الالماس فيه اعلى conductivity

Convection

Is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.



Newton's law of cooling

$$Q_{conv} = A_s h (T_s - T_\infty) \quad [W]$$

convection heat transfer coefficient "h" $[W/m^2K]$

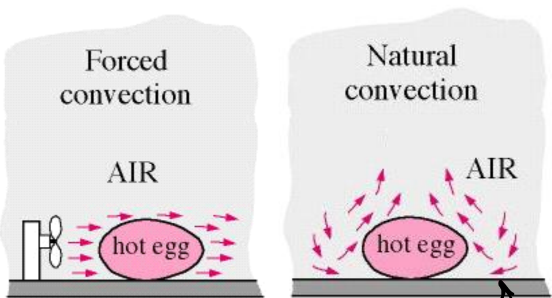
Depend on : Surface geometry
Type of fluid (liquid or gas)
Nature of fluid motion (المائع بيتحرك بانسيابية ولا مضطرب)
Fluid properties
bulk fluid velocity

Note : h is not fluid property.

Types of convection

→ Forced convection
the fluid molecules are forced to move by an external source

→ Free convection
the molecules move due to density and temp. variation



الهواء هيسخن فكثافته تقل فيطلع لفوق فيحل محله هواء بارد وهكذا

Radiation

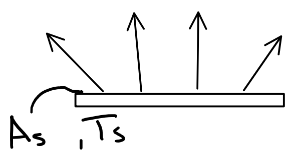
Is the energy emitted by matter in the form of electromagnetic waves.
Radiation does not require the presence of an intervening medium.
All bodies at a temp. above zero Kelvin emit thermal radiation.

Black-body

A hypothetical body that emit the maximum rate of radiation

$$Q_{emitted\ rad,max} = \sigma A_s \underbrace{T_s}_{[K]}^4 \quad [W]$$

stefan - Boltzmann const. $\sigma = 5.67 * 10^{-8}\ W/m^2K^4$



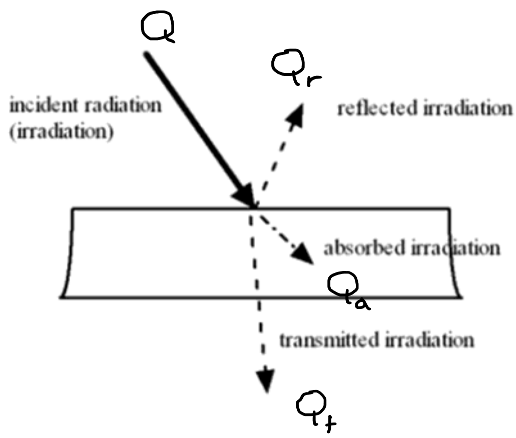
Real body

the radiation emitted by real body is less than the radiation emitted by black body

$$Q_{emitted\ rad} = \sigma \varepsilon A_s \underbrace{T_s}_{[K]}^4 \quad [W]$$

emissivity ε is a measure of how closely the real body to the black body $\varepsilon : (0 \to 1)$

When thermal radiation falls onto an object some combination of 3 things will happen



1. part of radiation will be absorbed

   $Q_{abs} = \alpha Q$ $\qquad$ α : absorbitivity

2. part of radiation will be transmitted

   $Q_{tran} = \tau Q$ $\qquad$ τ : transmissivity

3. part of radiation will be reflected

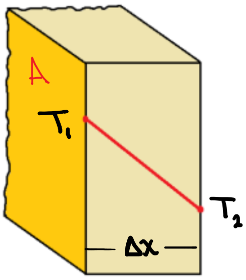
   $Q_{ref} = \rho Q$ $\qquad$ ρ : reflectivity

where $Q = Q_{abs} + Q_{ref} + Q_{tra}$

and $1 = \alpha + \rho + \tau$

Heat transfer modes

Conduction

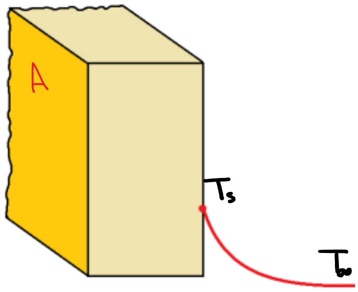


$$Q_{cond} = KA\frac{T_1 - T_2}{\Delta x} \quad [W]$$

Thermal conductivity "K" $\left[\frac{W}{mK}\right]$

$$q_{cond} = K\frac{T_1 - T_2}{\Delta x} \quad \left[\frac{W}{m^2}\right]$$

Convection

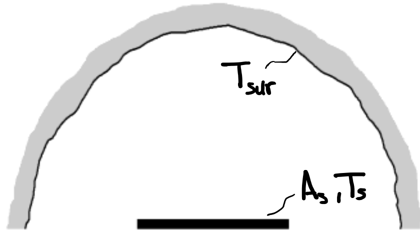


$$Q_{conv} = Ah(T_s - T_\infty) \quad [W]$$

Convection coeff "h" $\left[\frac{W}{m^2K}\right]$

$$q_{conv} = h(T_s - T_\infty) \quad \left[\frac{W}{m^2}\right]$$

Radiation

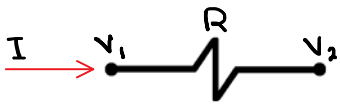


$$Q_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{sur}^4) \quad [W]$$

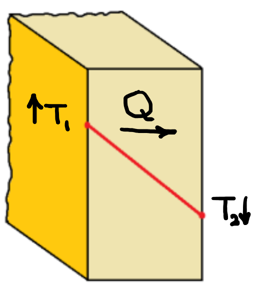
$$q_{rad} = \varepsilon\sigma(T_s^4 - T_{sur}^4) \quad \left[\frac{W}{m^2}\right]$$

Heat transfer and electrical analogy

Suppose we have a circuit in which current I is flowing. the resistance is R and the potential difference is ΔV. So by ohm's law $I = \frac{\Delta V}{R}$.



And if we compare the Fourier's law and ohm's law, then we see the similarties between them.



Electric current I ⟶ Heat flow rate Q

potential difference ΔV ⟶ Temperature difference ΔT

Electric resistance R ⟶ Thermal resistance

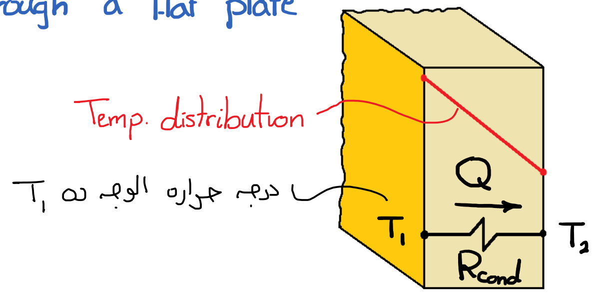
$$\& \quad Q = \frac{\Delta T}{R}$$

Conduction thermal resistance through a flat plate

$$Q_{cond} = KA\frac{T_1 - T_2}{\Delta x} = \frac{\Delta T}{R_{cond}}$$

$$R_{cond} = \frac{\Delta x}{AK} \quad \left[\frac{K}{W}\right]$$

$$R_{cond} = \frac{\Delta x}{K} \quad \left[\frac{Km^2}{W}\right]$$

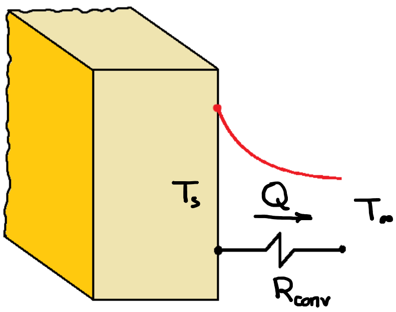


Convection thermal resistance

$$Q_{conv} = Ah(T_s - T_\infty) = \frac{\Delta T}{R_{conv}}$$

$$R_{conv} = \frac{1}{Ah} \quad \left[\frac{K}{W}\right]$$

$$R_{conv} = \frac{1}{h} \quad \left[\frac{Km^2}{W}\right]$$

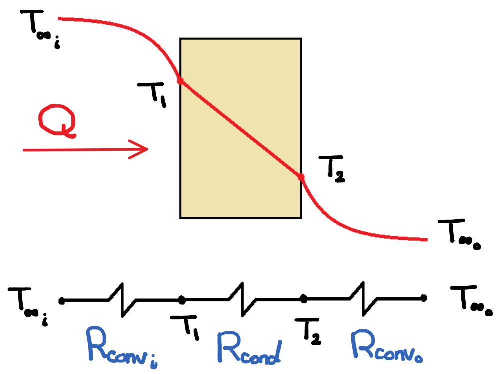


Single plane wall

$$R_{conv_i} = \frac{1}{Ah_i} \quad , \quad R_{conv_o} = \frac{1}{Ah_o}$$

$$R_{cond} = \frac{\Delta x}{AK}$$

$$Q = \frac{\Delta T}{\sum R}$$



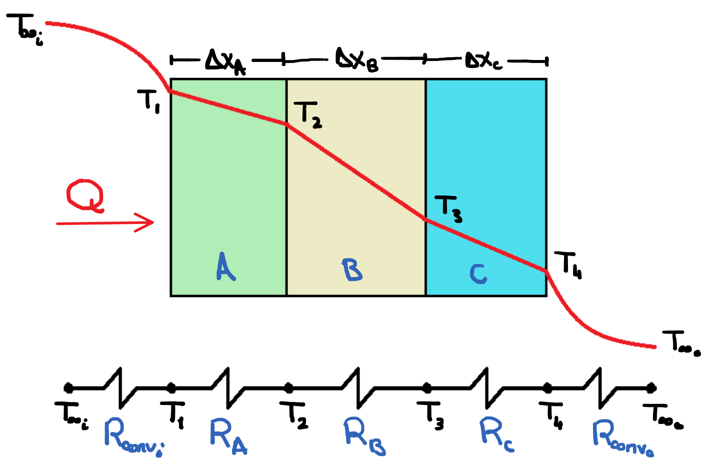
Composite plane wall

$$R_{conv_i} = \frac{1}{Ah_i} \quad , \quad R_{conv_o} = \frac{1}{Ah_o}$$

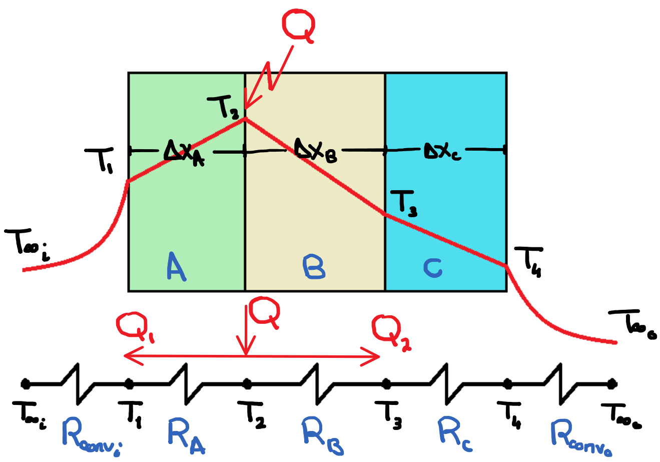
$$R_A = \frac{\Delta x_A}{AK_A} \quad , \quad R_B = \frac{\Delta x_B}{AK_B} \quad , \quad R_C = \frac{\Delta x_C}{AK_C}$$

$$Q = \frac{\Delta T}{\sum R}$$

$$= \frac{T_{\infty_i} - T_{\infty_o}}{R_{conv_i} + R_A + R_B + R_C + R_{conv_o}}$$



If heater is inserted between layer A and B



$$R_{conv_i} = \frac{1}{A h_i} \quad , \quad R_{conv_o} = \frac{1}{A h_o}$$

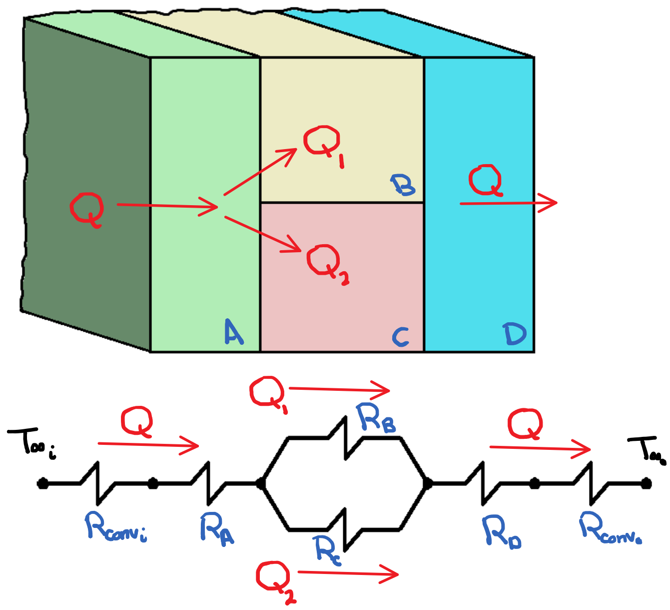
$$R_A = \frac{\Delta x_A}{A K_A} \quad , \quad R_B = \frac{\Delta x_B}{A K_B} \quad , \quad R_c = \frac{\Delta x_c}{A K_c}$$

$$Q = Q_1 + Q_2$$

$$Q_1 = \frac{\Delta T}{\sum R} = \frac{T_2 - T_{\infty_i}}{R_{conv_i} + R_A}$$

$$Q_2 = \frac{\Delta T}{\sum R} = \frac{T_2 - T_{\infty_o}}{R_B + R_c + R_{conv_o}}$$

In case of two parallel layers



$$R_{conv_i} = \frac{1}{A h_i} \quad , \quad R_{conv_o} = \frac{1}{A h_o}$$

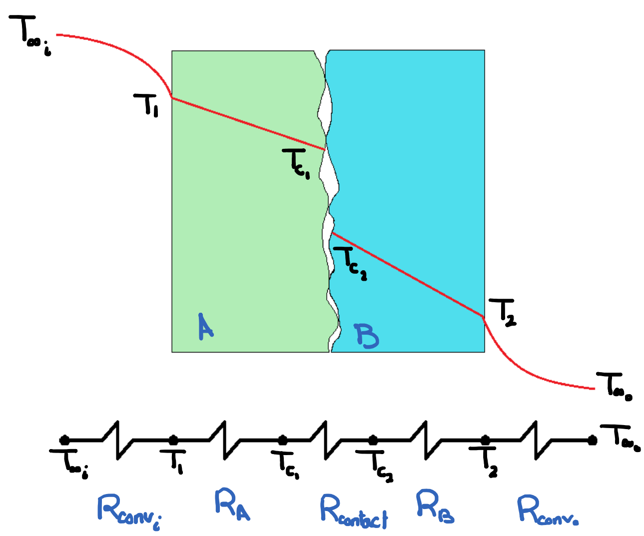
$$R_A = \frac{\Delta x_A}{A_A K_A} \quad , \quad R_B = \frac{\Delta x_B}{A_B K_B}$$

$$R_c = \frac{\Delta x_c}{A_c K_c} \quad , \quad R_D = \frac{\Delta x_D}{A_D K_D}$$

Contact resistance

the thermal contact resistance arises due to the improper contact between two bodies.

$R_{contact}$ usually given in $\left[\frac{Km^2}{W}\right]$
devide it by area to obtain it in $\left[\frac{K}{W}\right]$



Heat diffusion equation

it is a partial differential equation that describes heat distribution in a given body.

هى معادله بتوصف توزيع درجة الحراره جوه جسم .

$$\underbrace{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}+q=\rho c_p\frac{\partial T}{\partial t}$$

بيمثلوا توزيع درجات الحراره فى التلات اتجاهات (x, y, z)

heat generated: و ال term ده هيكون موجود لو فى حراره بتتولد جوه الجسم اللى بدرسه .

شوية امثلة :

- لو الجسم متوصل بتيار كهربى ساعتها هيتولد حراره .
- لو بيحصل تفاعل كيميائى طارد للحراره جوه الجسم .

ال term ده بيمثل تغير درجة الحراره مع الزمن واللى هيكون بصفر لو انا شغال عند ال steady state $\frac{\partial T}{\partial t}=0$

The thermal conductivity is a function of temperature, but if we assume it is constant, the general form of the heat diffusion equation could be simplified.

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+q=\rho c_p\frac{\partial T}{\partial t}$$

Divide by K:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

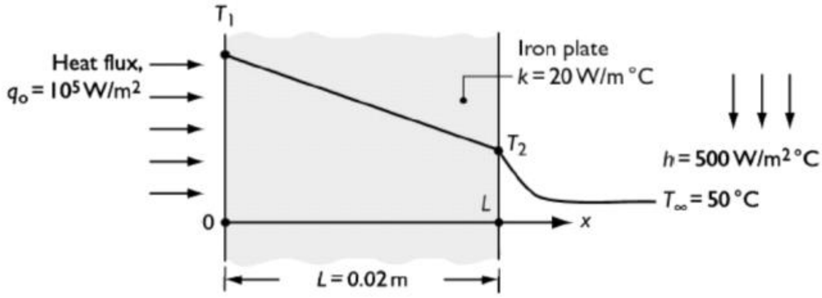
where α is the thermal diffusivity $\alpha=\frac{K}{\rho C_p}$ $[m^2/s]$

it measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy

فى مسائل محلوله هتفهم منها بستخدم القانون النهائى ده ازاى

Sheet (1)
One Dimension Steady-State Conduction Heat Transfer

1. An iron plate of thickness L and thermal conductivity k is subjected to a constant heat flux q_0 W/m^2 at the boundary surface at $x = 0$. From the other boundary surface at $x = L$, the heat is dissipated by convection into a fluid at temperature T_∞ with a heat transfer coefficient h. Develop the expressions for the surface temperatures T_1 and T_2 at the surfaces $x = 0$ and $x = L$ respectively. For the following data, calculate the surface temperatures T_1 and T_2 if $L = 2$ cm, $k = 20$ W/m °C, $q_0 = 105$ W/m^2, $T_\infty = 50$°C and $h = 500$ W/m^2 ^{0}C.



This problem could be solved by two methods { → using the heat diffusion eq. → using the thermal circuit

① By using the heat diffusion eq.

In our problem heat transfer is :

- one dimension $\frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial z^2} = 0$
- no heat generation $q = 0$
- Steady state $\frac{\partial T}{\partial t} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \cancel{\frac{q}{k}} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

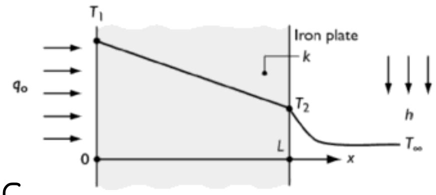
$$\therefore \frac{d^2 T}{d x^2} = 0 \xrightarrow{\text{integrate}} \frac{dT}{dx} = C_1 \xrightarrow{\text{integrate}} T = C_1 x + C_2$$

المعادله دى هنقدر نجيب منها درجه الحراره عند اى مسافه x

لكن الدول لازم اعرف C_1, C_2 بكام .

هعرف اجيب الثوابت دى عن طريق ال boundary condition

Boundary condition



$\boxed{\text{at } x = 0}$ q_0 تنتقل عن طريق ال conduction

$\therefore$ $q_0 = -K \dfrac{dT}{dx} \longrightarrow \dfrac{dT}{dx} = -\dfrac{q_0}{K} = C_1$

$\boxed{\text{at } x = L}$ الحرارة تنتقل عن طريق ال Convection

$q_0 = h(T - T_\infty) \longrightarrow q_0 = h(C_1 x + C_2 - T_\infty)$

$\therefore$ $C_2 = \dfrac{q_0}{h} - C_1 x + T_\infty = \dfrac{q_0}{h} - C_1 L + T_\infty$

$\therefore$ The heat distribution equation $T = C_1 x + \dfrac{q_0}{h} - C_1 L + T_\infty$

$$= \frac{q_0}{K}(L - x) + \frac{q_0}{h} + T_\infty$$

$$= q_0\left(\frac{L-x}{K} + \frac{1}{h}\right) + T_\infty$$

Get T_1 and T_2

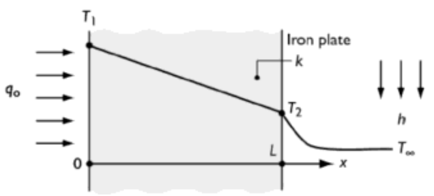
T_1 at $x=0$ $\therefore$ $T_1 = q_0\left(\dfrac{L}{K} + \dfrac{1}{h}\right) + T_\infty$

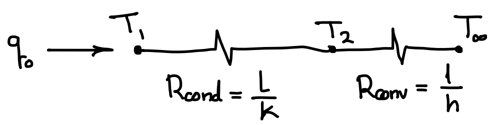
$$= 10^5\left(\frac{0.02}{20} + \frac{1}{500}\right) + 50 = 350℃$$

T_2 at $x = L$ $\therefore$ $T_2 = \dfrac{q_0}{h} + T_\infty$

$$= 10^5\left(\frac{1}{500}\right) + 50 = 250℃$$

② By using the thermal circuit

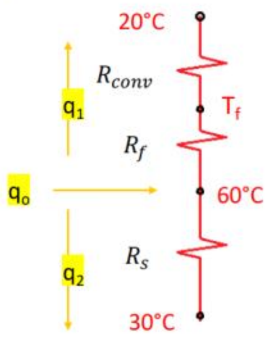


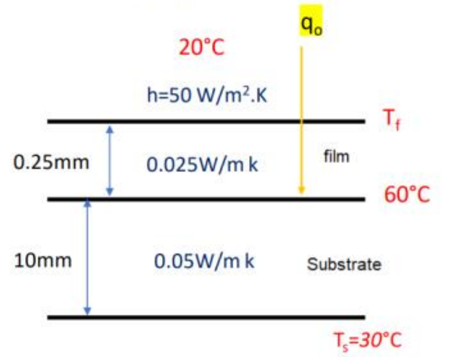


$q_0 = \dfrac{T_1 - T_\infty}{\frac{L}{K} + \frac{1}{h}}$ $\therefore$ $T_1 = q_0\left(\dfrac{L}{K} + \dfrac{1}{h}\right) + T_\infty = 350℃$

$q_0 = \dfrac{T_2 - T_\infty}{1/h}$ $\therefore$ $T_2 = \dfrac{q_0}{h} + T_\infty = 250℃$

1- In a manufacturing process, a transparent film is being bonded to a substrate as shown in the figure. To achieve the bond at a temperature T_o, a radiant source is used to provide a radiation heat flux q_o (W/m²), all of which is absorbed at the bonded surface. The back of the surface is maintained at T_s, while the free surface of the film is exposed to air at T_∞ and connection coefficient h. show the thermal circuit representing the steady-state heat transfer and calculate the value of q_o required to maintain the bonded surface at T_o=60°C and T_s=30°C. Take L_f=0.25mm, L_s=10mm, k_f=0.025, k_s=0.05W/m².°K, T_∞=20°C and h=50 W/m².K. Also calculate and T_f.





$$R_{conv} = \frac{1}{Ah} = \frac{1}{h} = \frac{1}{50} = 0.02\ k.m^2/w$$

$$R_f = \frac{\Delta x_f}{A.K_f} = \frac{\Delta x_f}{K_f} = \frac{0.00025}{0.025} = 0.01\ k.m^2/w$$

$$R_s = \frac{\Delta x_s}{A.K_s} = \frac{\Delta x_s}{K_s} = \frac{0.01}{0.05} = 0.2\ k.m^2/w$$

$$q_o = q_1 + q_2$$

$$q_1 = \frac{60-20}{R_{conv}+R_f} = \frac{40}{0.02+0.01} = 1333.3\ W/m^2$$

$$q_2 = \frac{60-30}{R_s} = \frac{30}{0.2} = 150\ W/m^2$$

$$q_o = 1333.3 + 150 = 1483.3\ W/m^2$$

$$q_1 = \frac{60-T_f}{R_f} \longrightarrow T_f = 46.67°C$$

If the film layer is not transparent and all the radiation heat flux is absorbed at its upper surface, determine the radiation heat flux required to achieve bonding

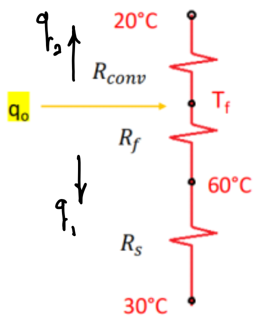
$$q_{r_1} = \frac{60-30}{0.2} = 150\ W/m^2$$

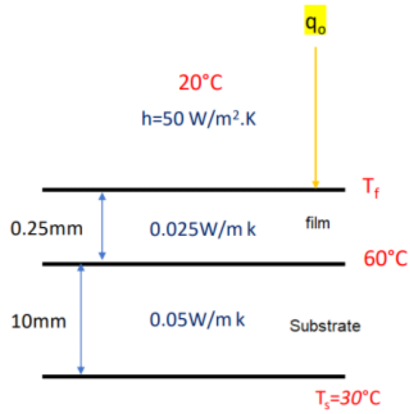
$$= \frac{T_f - 60}{0.01}$$

$$\therefore T_f = 61.5°C$$

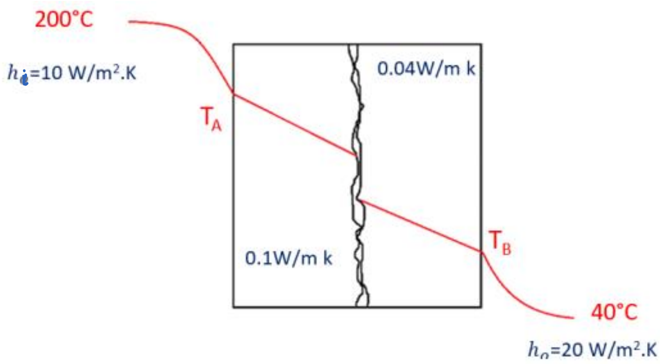
$$q_{r_2} = \frac{T_f - 20}{0.02} = 2075\ W/m^2$$

$$\therefore q_{r_o} = q_{r_1} + q_{r_2} = 2225\ W/m^2$$





6. Consider a plane composite wall is composed of two materials (A &B) of thermal conductivity k_A=0.1 W/m.K and k_B=0.04 W/m.K and thickness L_A=10mm and L_B=20mm. The contact resistance at the interface between the two materials is known to be 0.3m^2.K/W. Material A adjoins a fluid at 200°C for which h=10 W/m^2.K, while material B adjoins a fluid at 40°C for which h=20 W/m^2.K; sketch the temperature distribution and calculate the heat transfer rate through the wall if it is 2.5m long &2m wide. Also calculate the outer surface temperature for material A&B and the drop in temperature across the interface.



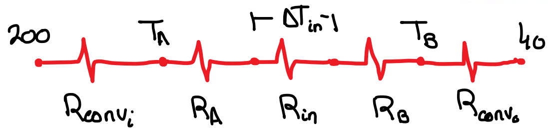
$$R_{conv_i} = \frac{1}{Ah_i} = \frac{1}{5*10} = 0.02\ \frac{K}{W}$$

$$R_A = \frac{\Delta X_A}{AK_A} = \frac{0.01}{5*0.1} = 0.02\ \frac{K}{W}$$

$$R_{in} = \frac{\bar{R}_{in}}{A} = \frac{0.3}{5} = 0.06\ \frac{K}{W}$$

$$R_B = \frac{\Delta X_B}{AK_B} = \frac{0.02}{5*0.04} = 0.1\ \frac{K}{W}$$

$$R_{conv_o} = \frac{1}{Ah_o} = \frac{1}{5*20} = 0.01\ \frac{K}{W}$$



$$Q = \frac{200-40}{R_{conv_i} + R_A + R_{in} + R_B + R_{conv_o}} = 761.9\ W$$

$$Q = \frac{200 - T_A}{R_{conv_i}} \qquad \& \quad T_A = 184.76\ ^\circ C$$

$$Q = \frac{T_B - 40}{R_{conv_o}} \qquad \& \quad T_B = 47.62\ ^\circ C$$

$$Q = \frac{\Delta T_{in}}{R_{in}} \qquad \& \quad \Delta T_{in} = 45.714\ ^\circ C$$

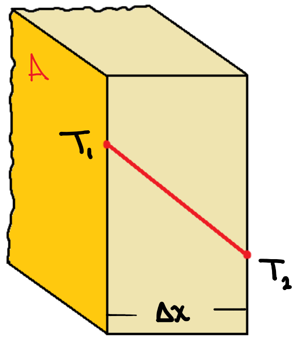
One dimension steady state conduction heat transfer Cont.

Conduction – Fourier's law of heat conduction $\dot{Q}_{cond} = -KA\frac{dT}{dx}$

← يتمثل المسافة اللى هيتنقل فيها الـ Q

For a plane wall

Area "A" = Const.



$$\dot{Q}_{cond}\,dx = -KA\,dT$$

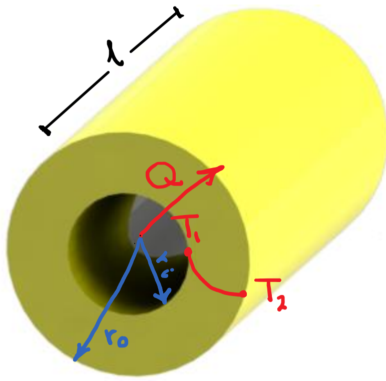
by integration :

$$\int_{x_1}^{x_2}\dot{Q}_{cond}\,dx = \int_{T_1}^{T_2}-KA\,dT$$

$$\therefore\ \dot{Q}_{cond} = KA\frac{T_1-T_2}{\Delta x}$$

For a cylinderical wall

$A = 2\pi r l$



$$\dot{Q}_{cond} = -KA\frac{dT}{dr}$$

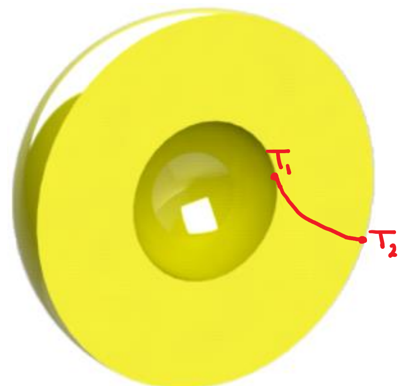
$$\int_{r_i}^{r_o}\dot{Q}_{cond}\frac{dr}{r} = \int_{T_1}^{T_2}-K(2\pi l)\,dT$$

$$\dot{Q}_{cond}\ \ln\frac{r_o}{r_i} = 2\pi l K(T_1-T_2)$$

$$\dot{Q}_{cond} = \frac{2\pi l K}{\ln r_o/r_i}(T_1-T_2)$$

For a spherical wall

$A = 4\pi r^2$



$$\dot{Q}_{cond} = -KA\frac{dT}{dr}$$

$$\int_{r_i}^{r_o}\dot{Q}_{cond}\frac{dr}{r^2} = \int_{T_1}^{T_2}-K\cdot 4\pi\,dT$$

$$\dot{Q}_{cond}\ \left.\frac{-1}{r}\right|_{r_i}^{r_o} = 4\pi K(T_1-T_2)$$

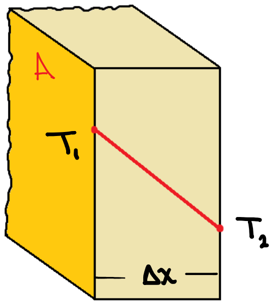
$$\dot{Q}_{cond} = \frac{4\pi K}{\frac{1}{r_i}-\frac{1}{r_o}}(T_1-T_2)$$

ملاحظة مهمة : الـ Temp. distribution في الاسطوانة و الكورة بيبقى على شكل منحنى مش خط مستقيم زى الـ plane wall وده بسبب ان الـ A بتكبر كل ماتتحرك للخارج وبالتالى الميل $slope = \frac{dT}{dx}$ لازم يقل علشان تفضل الـ Q بقيمة ثابتة بما ان شغلنا هنا steady state

$$\boxed{Q = -KA\uparrow\frac{dT}{dx}\downarrow}$$

Conduction heat transfer

plane wall

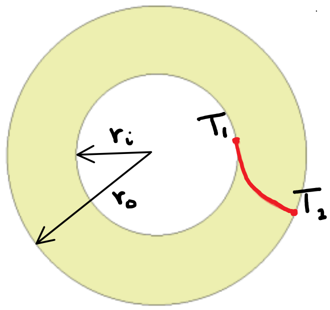


$$Q_{cond} = KA\frac{T_1 - T_2}{\Delta x}$$

$$R_{wall} = \frac{\Delta x}{AK} \quad \left[\frac{K}{W}\right]$$

$$R_{wall} = \frac{\Delta x}{K} \quad \left[\frac{Km^2}{W}\right]$$

Cylinderical wall

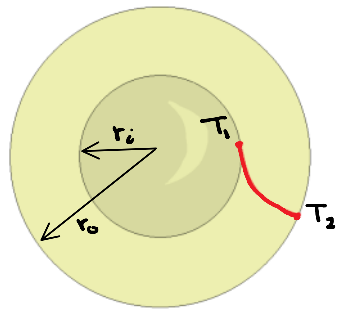


$$Q_{cond} = \frac{2\pi l K}{\ln r_o/r_i}(T_1 - T_2)$$

$$R_{cyl} = \frac{\ln r_o/r_i}{2\pi l K} \quad \left[\frac{K}{W}\right]$$

$$R_{cyl} = \frac{\ln r_o/r_i}{2\pi K} \quad \left[\frac{Km}{W}\right]$$

Spherical wall



$$Q_{cond} = \frac{4\pi K}{\frac{1}{r_i} - \frac{1}{r_o}}(T_1 - T_2)$$

$$R_{sph} = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi K} \quad \left[\frac{K}{W}\right]$$

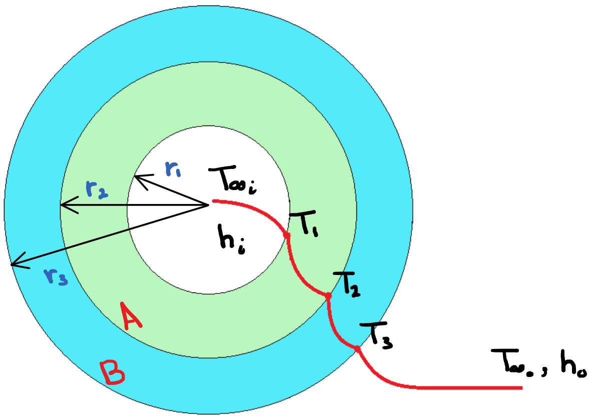
Composite cylinderical wall

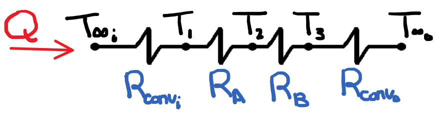
$$R_{conv_i} = \frac{1}{A_1 h_i} = \frac{1}{2\pi r_1 l h_i}$$

$$R_A = \frac{\ln r_2/r_1}{2\pi l K_A} \quad , \quad R_B = \frac{\ln r_3/r_2}{2\pi l K_B}$$

$$R_{conv_o} = \frac{1}{A_3 h_o} = \frac{1}{2\pi r_3 l h_o}$$

$$Q = \frac{\Delta T}{\Sigma R}$$





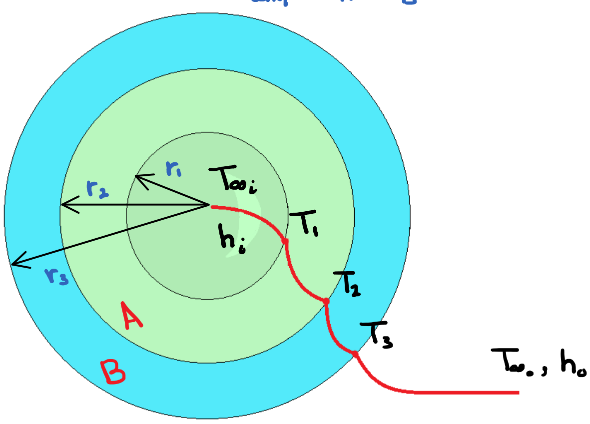
Composite spherical wall

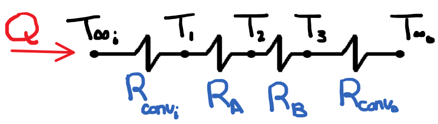
$$R_{conv_i} = \frac{1}{A_1 h_i} = \frac{1}{4\pi r_1^2 h_i}$$

$$R_A = \frac{1/r_1 - 1/r_2}{4\pi K_A} \quad , \quad R_B = \frac{1/r_2 - 1/r_3}{4\pi K_B}$$

$$R_{conv_i} = \frac{1}{A_3 h_i} = \frac{1}{4\pi r_3^2 h_o}$$

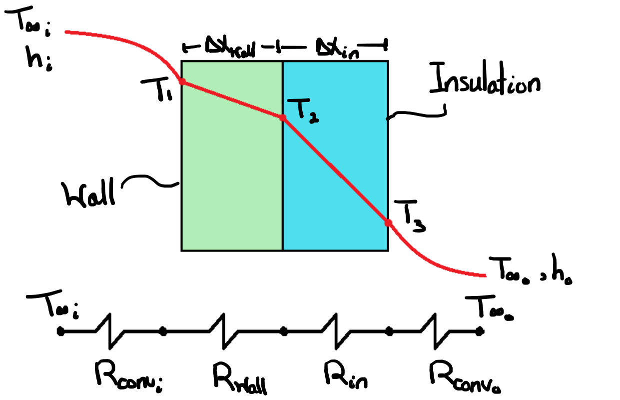
$$Q = \frac{\Delta T}{\Sigma R}$$





Adding insulation

For a plane wall



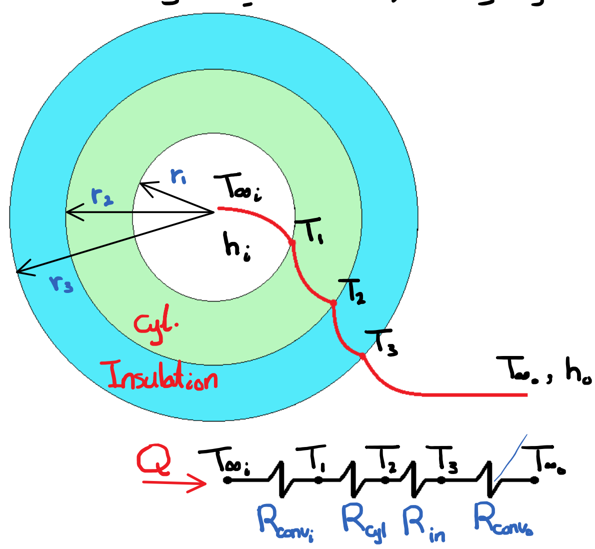
$$R_{conv_i} = \frac{1}{A h_i} \quad , \quad R_{conv_o} = \frac{1}{A h_o}$$

$$R_{wall} = \frac{\Delta x_{wall}}{A K_{wall}} \quad , \quad R_{in} = \frac{\Delta x_{in}}{A K_{in}}$$

$$Q = \frac{\Delta T}{\sum R} = \frac{T_{\infty_i} - T_{\infty_o}}{R_{conv_i} + R_{wall} + R_{in} + R_{conv_o}}$$

زيادة سمك العزل Δx_{in} .يزود قيم مقاومة العزل R_{in} وبكده $R_{total} = \sum R$ هتزيد وبكده Q تقل
وهو ده الهدف من عملية العزل .

For a cylinderical wall



$$R_{conv_i} = \frac{1}{A_1 h_i} = \frac{1}{2\pi r_1 L h_i}$$

$$R_{cyl} = \frac{\ln r_2/r_1}{2\pi L K_{cyl}} \quad , \quad R_{in} = \frac{\ln r_3/r_2}{2\pi L K_{in}}$$

$$R_{conv_o} = \frac{1}{A_3 h_o} = \frac{1}{2\pi r_3 L h_o}$$

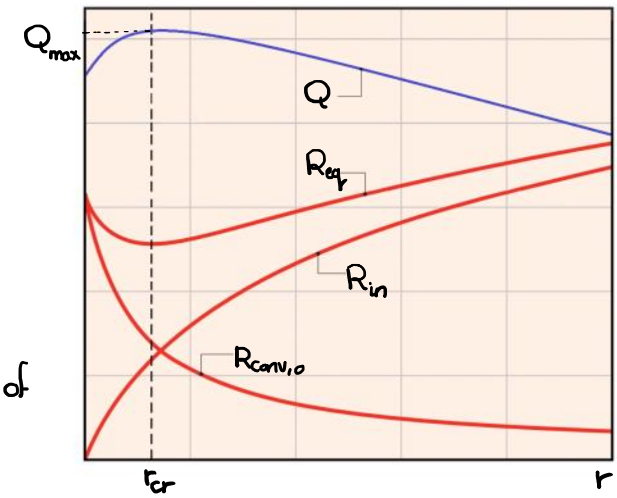
زيادة سمك طبقة العزل معناه زيادة r_3 وده هيزود
قيم R_{in} و هيقلل R_{conv_o} .

$$R_{eq} = \frac{\ln r_3/r_2}{2\pi L K_{in}} + \frac{1}{2\pi r_3 L h_o}$$

Differentiate and equate to zero to get the radius at which Req is minimum.

$$\frac{dR_{eq}}{dr_3} = \frac{1}{\cancel{2\pi L} K_{in} r_3} - \frac{1}{\cancel{2\pi L} h_o r_3^2} = 0$$

$\therefore$ Critical radius $r_{cr} = \dfrac{K_{in}}{h_o}$



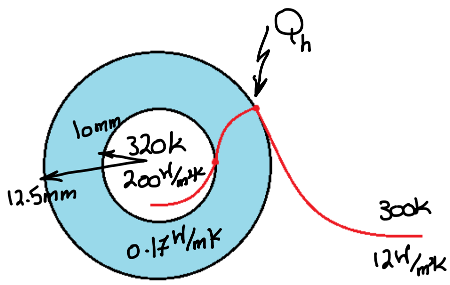
- For an effictive insulation the outer radius of the insulation should be more than r_{cr} .
- to make sure any thickness of an insulation is effective, the insulation conductivity must be equal or less than $K_{in,max} = r_o h_o$ where r_o is the bare pipe radius.
- In case of spherical wall $r_{cr} = \dfrac{2K_{in}}{h_o}$.

7. Water at an average temperature of 320K flows inside a Teflon tube (k=0.17W/m.K) with inside connective coefficient h_i=200W/m^2.K. The inner and outer diameters of the tube are 20mm and 25mm respectively. A thin electric heating tape is wounded around the tube. The tape provides a uniform heat flux of 2kW/m^2, and ambient air at temperature of 300K, maintains an outside convective coefficient of 12W/m^2.K. Calculate:

i- The outer surface temperature of the Teflon tube.

ii- The percentage of heat flux transferred to the water.

[Ans. i- T_h=351.4K and ii.69.3%].

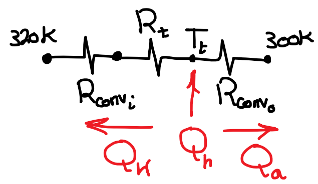


$$Q_h = 2000\ W/m^2 = 2000 * \pi * 0.025 = 157.08\ W/m$$

$$R_{conv_i} = \frac{1}{2\pi r_i h_i} = \frac{1}{2\pi * 0.01 * 200} = 0.0796\ \frac{Km}{W}$$

$$R_t = \frac{\ln r_o/r_i}{2\pi K_t} = \frac{\ln 12.5/10}{2\pi * 0.17} = 0.209\ \frac{Km}{W}$$

$$R_{conv_o} = \frac{1}{2\pi r_o h_o} = \frac{1}{2\pi * 0.0125 * 12} = 1.061\ \frac{Km}{W}$$



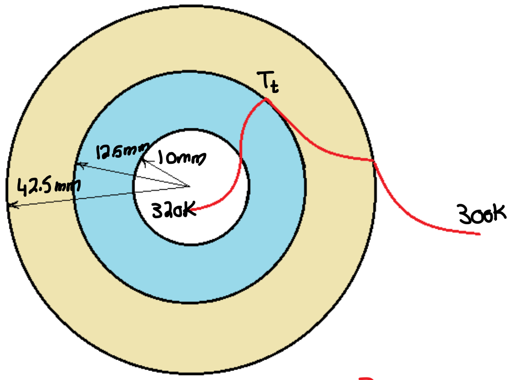
$$Q_h = Q_w + Q_a = \frac{T_t - 320}{R_{conv_i} + R_t} + \frac{T_t - 300}{R_{conv_o}} \quad \& \quad T_t = 351.4\ K$$

$$Q_w = \frac{T_t - 320}{R_{conv_i} + R_t} = 108.8\ W/m$$

the percentage of heat flux transfered to the water $= \frac{Q_w}{Q_h} = 0.693 = 69.3\ \%$

If we need benefit insulation around the electric heating tape in the previous problem to increase the percentage of heat that transferred to water, determine the maximum value of $k_{in,max}$ and repeat calculation of items i and ii when use 30 mm thickness for that insulation with the same heater capacity (2 kW/m^2).

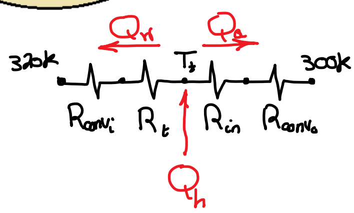
[Ans. $k_{in,max}$=0.15W/m.K, i-T_h= 355.4K and ii-78.3%]



$$K_{in,max} = r_o h_o = 0.0125 * 12 = 0.15\ W/mK$$

$$R_{in} = \frac{\ln r_o/r_i}{2\pi K_{in,max}} = \frac{\ln 42.5/12.5}{2\pi * 0.15} = 1.298\ \frac{Km}{W}$$

$$R_{conv_o} = \frac{1}{2\pi * 0.0425 * 12} = 0.312\ \frac{Km}{W}$$



$$Q_h = \frac{T_t - 320}{R_{conv_i} + R_t} + \frac{T_t - 300}{R_{in} + R_{conv_o}}$$

$$\& \quad T_t = 355.4\ K$$

$$Q_w = \frac{T_t - 320}{R_{conv_i} + R_t} = 122.66\ W/m$$

the percentage of heat flux $= \frac{Q_w}{Q_h} = 78\ \%$

10. A hollow aluminum sphere, with an electric heater in the center, is use to test to determine the thermal conductivity of insulating materials. The inner and outer radii of the sphere are 0.15m and 0.18m, respectively and testing is done under steady state conditions. With the inner surface of the aluminum maintained at 250°C in a particular test, a spherical shell of insulation is cast on the outer surface of the sphere to a thickness of 0.12m.The system in a room for which the air temperature is 20°C with convective coefficient 30W/m^2.K. If 80 Watt is dissipate by the heater under steady-state conditions. What the thermal conductivity of the insulation? ***[Ans. k_{in}=0.0622W/m.K].***

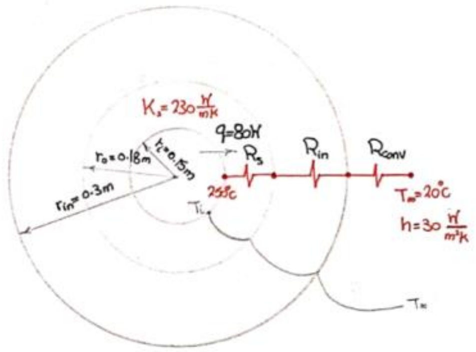
$$R_s = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi K_s} = \frac{\frac{1}{0.15} - \frac{1}{0.18}}{4\pi * 230} = 3.844 * 10^{-4} \ \frac{K}{W}$$

$$R_{in} = \frac{\frac{1}{r_o} - \frac{1}{r_{in}}}{4\pi K_{in}} = \frac{\frac{1}{0.18} - \frac{1}{0.3}}{4\pi K_{in}} = \frac{0.1768}{K_{in}}$$

$$R_{conv} = \frac{1}{A_{in} h} = \frac{1}{4\pi r_{in}^2 h} = \frac{1}{4\pi * 0.3^2 * 30} = 0.02947 \ \frac{K}{W}$$

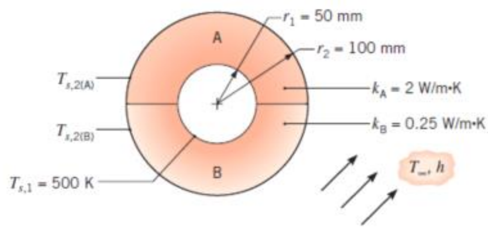
$$q = \frac{250 - 20}{R_s + R_{in} + R_{conv}} = \frac{230}{3.844*10^{-4} + \frac{0.1768}{K_{in}} + 0.02947} = 80$$

$$K_{in} = 0.06214 \ \frac{W}{mK}$$



11. Air flowing through a long, thin-walled pipe maintaining the inner wall at a uniform temperature of 500K. The pipe is covered with an insulation blanket composed of two different materials ***A*** and ***B*** as shown in the figure. The interface between the two materials may be assumed to have an infinite contact resistance and the entire outer surface is exposed to air for which T_{air}=300K and h=25 W/m^2.K. What is the total heat loss from the pipe? Also calculate Temperature of $T_{s2,A}$ and $T_{s2,B}$.

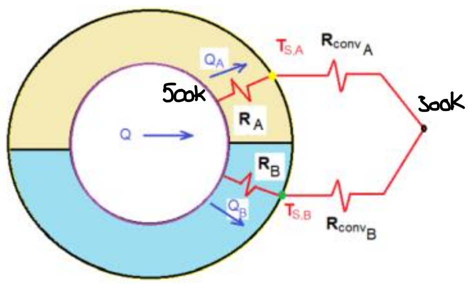
[Ans. Q_{Loss}=1040W/m.L, $T_{s,A}$=407K and $T_{s,B}$=325K]



$$R_A = 2R_{cyl} = 2\frac{\ln r_o/r_i}{2\pi K_A} = 2\frac{\ln 100/50}{2\pi * 2} = 0.1103 \ \frac{Km}{W}$$

$$R_B = 2R_{cyl} = 2\frac{\ln r_o/r_i}{2\pi K_B} = 2\frac{\ln 100/50}{2\pi * 0.25} = 0.8825 \ \frac{Km}{W}$$

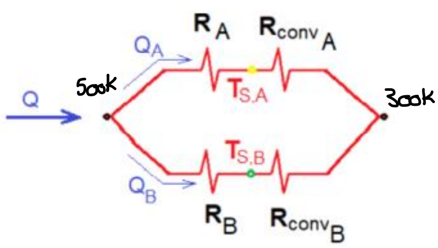
$$R_{conv_A} = R_{conv_B} = \frac{1}{\pi r_o h_o} = \frac{1}{\pi * 0.1 * 25} = 0.1273 \ \frac{Km}{W}$$



$$Q_A = \frac{500 - 300}{R_A + R_{conv_A}} = 842 \ W/m$$

$$Q_B = \frac{500 - 300}{R_B + R_{conv_B}} = 198 \ W/m$$

$$Q_{Loss} = Q_A + Q_B = 1040 \ W/m$$



$$Q_A = \frac{500 - T_{s,A}}{R_A} \quad \& \quad T_{s,A} = 407 \ K$$

$$Q_B = \frac{500 - T_{s,B}}{R_B} \quad \& \quad T_{s,B} = 325 \ K$$

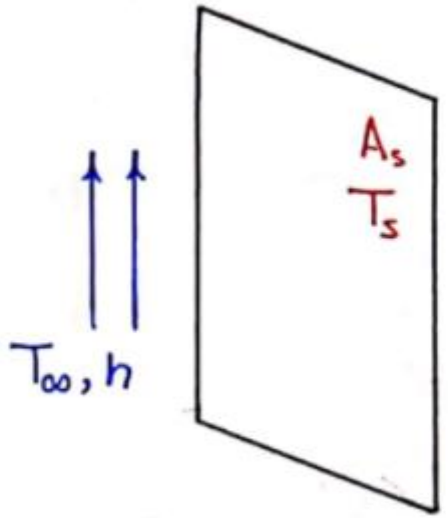
Heat Transfer

Forced convection Heat-Transfer
Part 1

Convection heat rate

$$q = A_s \, h \, \Delta T$$

h : Convection heat transfer coefficient

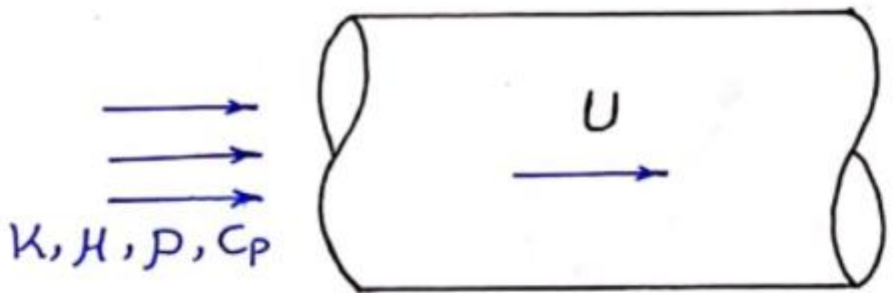


Convection heat transfer coefficient calculations

$$h = f\,(\, D_{eq} \,,\; U \,,\; \mu \,,\; \rho \,,\; C_p \,,\; k \,)$$

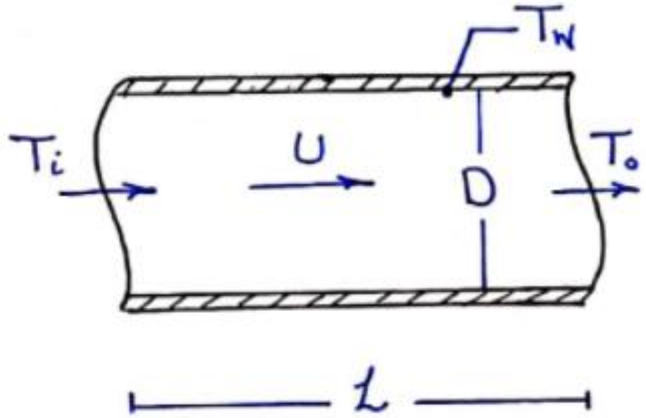
$$\frac{h\,D_{eq}}{k} = f\left(\frac{\rho\,U\,D_{eq}}{\mu} \,,\; \frac{\mu\,C_p}{k}\right)$$

$$Nu = f\,(\,Re \,,\; Pr\,)$$



Empirical Nusselt equations:

Flow through tubes or ducts:



A. For laminar flow $Re \leq 2300$

$$Nu = 1.86\left[Re \cdot Pr\right]^{1/3}\left[\frac{D_{eq}}{L}\right]^{1/3}\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

B. For turbulent flow $Re > 2300$

$$Nu = 0.023\, Re^{0.8}\, Pr^{n}$$

$n = 0.4$ for heating $n = 0.3$ for cooling

$$Re = \frac{\rho U D_{eq}}{\mu} \qquad Pr = \frac{\mu C_p}{K}$$

all properties are evaluated at mean bulk temperature

$$T_{mb} = \frac{T_i + T_o}{2}$$

Except μ_w at wall temperature

Equivalent diameter

$$D_{eq} = \frac{4A_c}{P_w}$$

① Circular pipe

$$D_{eq} = \frac{4\frac{\pi}{4}D^2}{\pi D} = D$$

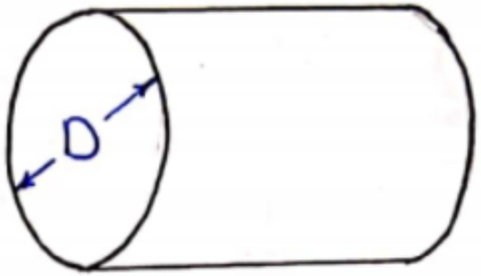
② Rectangular duct

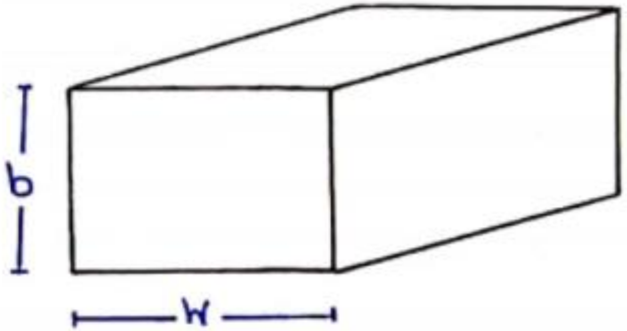
$$D_{eq} = \frac{4bW}{2(b+W)}$$

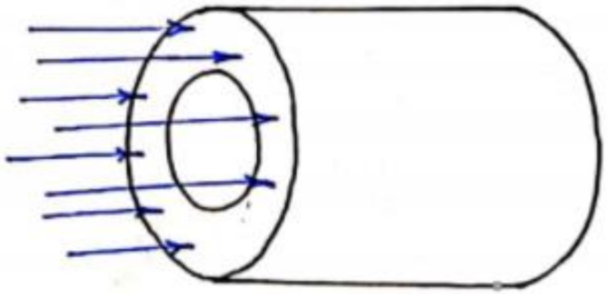
③ Annular tube

$$D_{eq} = \frac{4 \cdot \frac{\pi}{4}(D_o^2 - D_i^2)}{\pi(D_o + D_i)}$$

$$= D_o - D_i$$

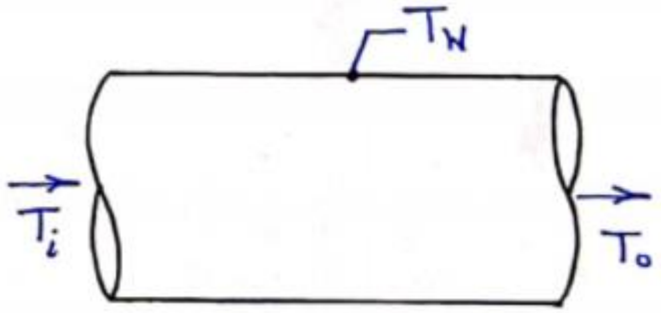






★ Solution steps :

- Get mean bulk temperature $T_{mb} = \frac{T_i + T_o}{2}$



- Get ρ, C, μ, K and Pr from the fluid table at T_{mb}

TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m^3)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
600	0.5804	1.051	305.8	52.69	46.9	76.9	0.685
650	0.5356	1.063	322.5	60.21	49.7	87.3	0.690
700	0.4975	1.075	338.8	68.10	52.4	98.0	0.695
750	0.4643	1.087	354.6	76.37	54.9	109	0.702
800	0.4354	1.099	369.8	84.93	57.3	120	0.709

TABLE A.6 Thermophysical Properties of Saturated Water[a]

Temperature, T (K)	Pressure, p (bars)[b]	Specic Volume (m^3/kg) $v \cdot 10^3$	v_g	Heat of Vaporization, h_{g} (kJ/kg)	Specic Heat ($kJ/kg \cdot K$) $c_{p,}$	$c_{p,g}$	Viscosity ($N \cdot s/m^2$) $\mu \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity ($W/m \cdot K$) $k \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr	Pr_g	Surface Tension, $\sigma \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta \cdot 10^6$ (K^{-1})
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	−68.05
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	−32.74
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174.0
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865	70.9	320.6
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873	70.0	361.9
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883	69.2	400.4
320	0.1053	1.011	13.98	2390	4.180	1.895	577	9.89	640	21.0	3.77	0.894	68.3	436.7
325	0.1351	1.013	11.06	2378	4.182	1.903	528	10.09	645	21.3	3.42	0.901	67.5	471.2
330	0.1719	1.016	8.82	2366	4.184	1.911	489	10.29	650	21.7	3.15	0.908	66.6	504.0
335	0.2167	1.018	7.09	2354	4.186	1.920	453	10.49	656	22.0	2.88	0.916	65.8	535.5
340	0.2713	1.021	5.74	2342	4.188	1.930	420	10.69	660	22.3	2.66	0.925	64.9	566.0

- $Re = \frac{\rho U D_{eq}}{\mu}$

Choose the appropriate Nusselt equation

$Re \leq 2300$ Laminar flow

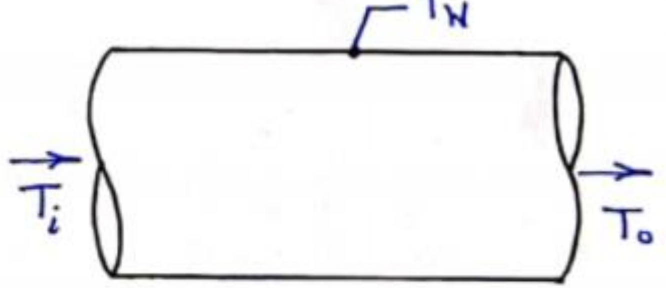
$Re > 2300$ Turbulent flow

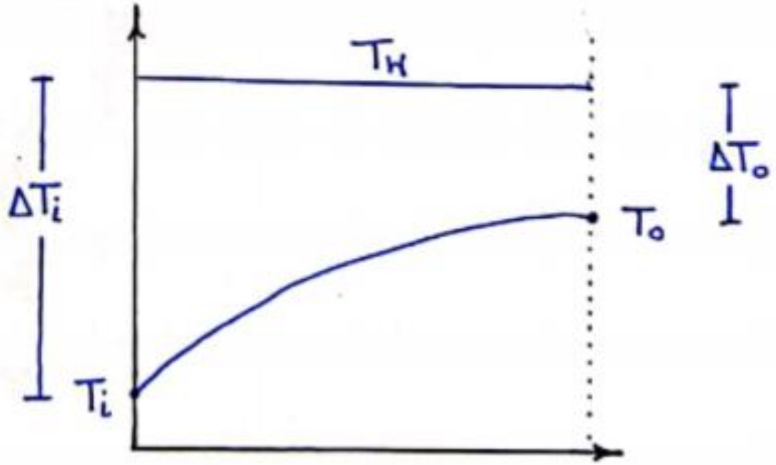
- Calculate the Nusselt no.

- Calculate the convection coefficient $Nu = \frac{h D_{eq}}{K}$

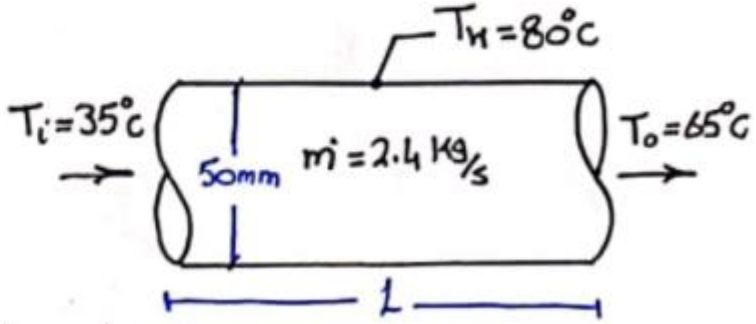
- Energy balance

$$q = A_s h \left(T_w - T_{mb} \right) = \dot{m} C \left(T_o - T_i \right)$$





1-Water flows inside a tube 50mm diameter at mass flow rate of 2.4 kg/s. The water is to be heated from 35°C to 65°C by maintaining the tube surface temperature at 80°C. How long must the tube be to accomplish this heating?



Notes that: May you use the correlation equation for fluid flows through ducts as follows:

For turbulent flow where Re>2300

$$Nu = 0.023 Re^{0.8} Pr^{n} \qquad \text{where } n=0.3 \text{ for cooling and } n=0.4 \text{ for heating.}$$

For laminar flow Re≤ 2300

$$Nu = 1.86 [Re.Pr]^{1/3} [D/L]^{1/3} [\mu/\mu_w]^{0.14}$$

for both laminar and turbulent forced convection the properties are evaluated at mean bulk temperature Tm_b, except μ_w at wall temperature Tw.

- Determine the length of the tube

mean bulk temperature

$$T_{mb} = \frac{T_i + T_o}{2} = \frac{65+35}{2} = 50°c$$

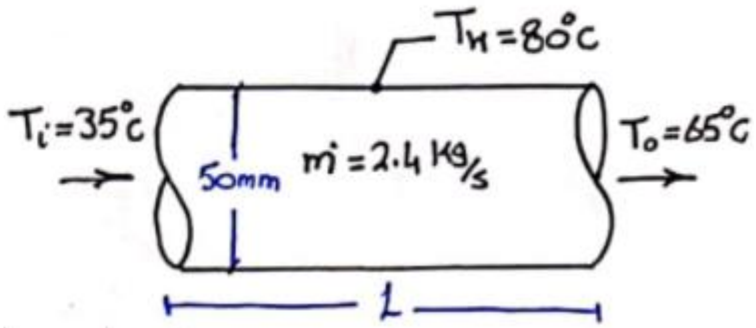
TABLE A.6 Thermophysical Properties of Saturated Water[a]

Temperature, T (K)	Pressure, p (bars)[b]	Specic Volume (m³/kg) $v_f \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specic Heat (kJ/kg·K) $c_{p,f}$	$c_{p,g}$	Viscosity (N·s/m²) $\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m·K) $k_f \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr_f	Pr_g	Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefcient, $\beta_f \cdot 10^6$ (K⁻¹)
320	0.1053	1.011	13.98	2390	4.180	1.895	577	9.89	640	21.0	3.77	0.894	68.3	436.7
325	0.1351	1.013	11.06	2378	4.182	1.903	528	10.09	645	21.3	3.42	0.901	67.5	471.2

$\rho = 988 \frac{Kg}{m^3}$ $\quad C = 4182 \frac{J}{Kg.K}$ $\quad \mu = 544 * 10^{-6} \frac{Kg}{m.s}$ $\quad K = 0.643 \frac{W}{mK}$ $\quad Pr = 3.54$

$$Re = \frac{\rho U D}{\mu}$$

$$\dot{m} = \rho A_c U = 988 * \frac{\pi}{4} * 0.05^2 U = 2.4 \longrightarrow U = 1.237 \ m/s$$



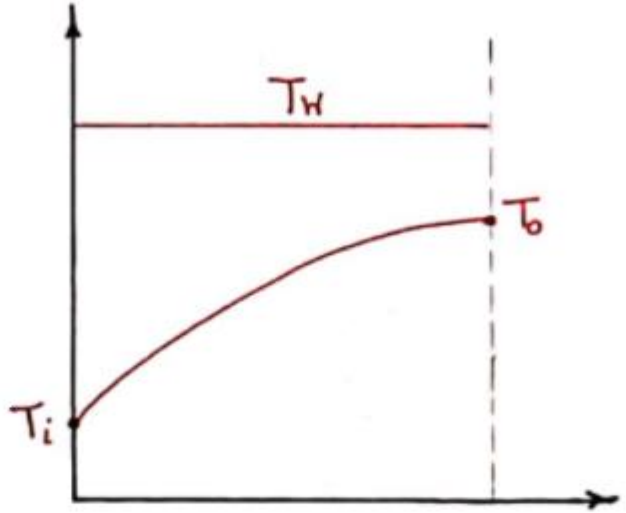
$$Re = \frac{\rho U D}{\mu} = \frac{988 * 1.237 * 0.05}{544 * 10^{-6}} = 112330.5 \quad > 2300$$

$\therefore$ The flow is turbulent $\quad n = 0.4$ for heating

$$Nu = 0.023 \, Re^{0.8} \, Pr^{0.4} = 0.023 * 112330.5^{0.8} * 3.54^{0.4} = 418.53$$

$$Nu = \frac{hD}{K} = \frac{0.05h}{0.643}$$

$$\therefore h = 5382.3 \ \frac{W}{m^2K}$$

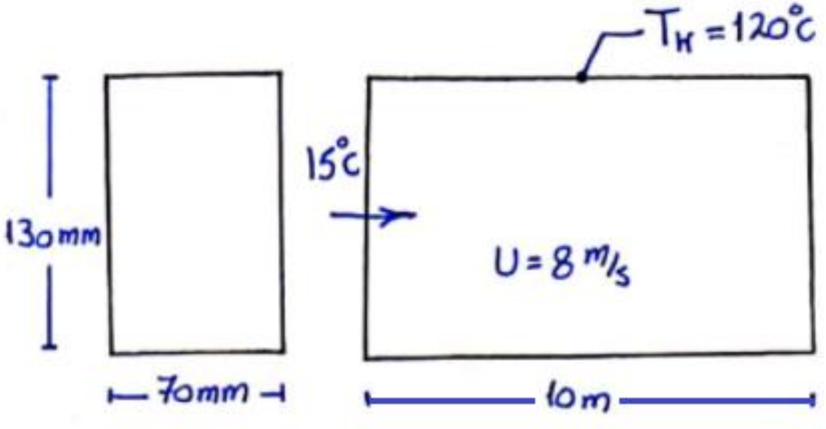


$$q = \dot{m} C (T_o - T_i) = A_s h (T_H - T_{mb})$$

$$2.4 * 4182 * (65 - 35) = \pi * 0.05 \, L * 5382.3 \, (80 - 50)$$

$$\boxed{L = 11.87 \ m}$$

5-Air at 15°C and 1 atmosphere flows through a long rectangular duct with dimensions of 70mmx130mm. The surface temperature of the duct wall is maintained at 120°C. If the mean air velocity through the duct is 8m/s calculate the heat transfer rate and the air exit temperature.



***Notes that*: May you use the correlation equation for fluid flows through ducts as follow**

For turbulent flow where Re>2300

$$Nu = 0.023Re^{0.8}\, Pr^{n} \qquad \text{where } n=0.3 \text{ for cooling and } n=0.4 \text{ for heating.}$$

For laminar flow Re≤ 2300

$$Nu = 1.86[Re.Pr]^{1/3}\,[D/L]^{1/3}[\mu/\mu_w]^{0.14}$$

for both laminar and turbulent forced convection the properties are evaluated at mean bulk temperature Tm_b, except μ_w at wall temperature Tw.

where $D_{eq}=4A_c/P$

First trail : assume $T_o = 89°C$

$$T_{mb} = \frac{T_i + T_o}{2} = 52°C = 325\,K$$

$\rho = 1.086 \frac{Kg}{m^3}$ $\quad C = 1006.3 \frac{J}{Kg.K}$

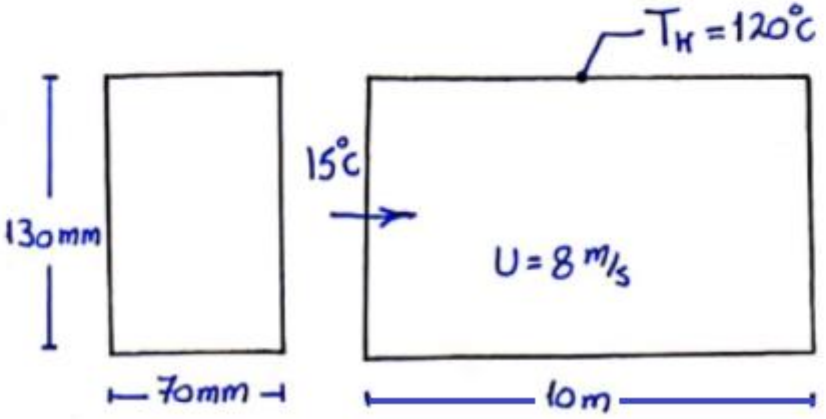
$\mu = 1.962 * 10^{-5} \frac{Kg}{m.s}$ $\quad K = 0.02816 \frac{W}{mK}$

$Pr = 0.701$

TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m²)	$\nu \cdot 10^6$ (m²/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m²/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690

$$D_{eq} = \frac{4A_c}{P_w} = \frac{4 * 0.07 * 0.13}{2(0.13+0.07)} = 0.091 \text{ m}$$



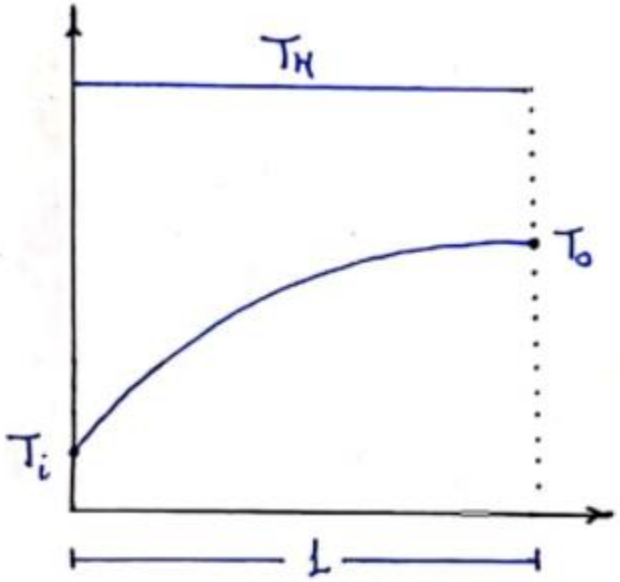
$$Re = \frac{\rho U D_{eq}}{\mu} = \frac{1.086 * 8 * 0.091}{1.962 * 10^{-5}} = 40296$$

$Re > 2300$ the flow is turbulent

$$Nu = 0.023 \, Re^{0.8} \, Pr^{0.4} = 0.023 * 40296^{0.8} * 0.701^{0.4} = 96.43$$

$$= \frac{h D_{eq}}{k} = \frac{0.091 h}{0.02816}$$

$$h = 29.84 \ \frac{W}{m^2 K}$$



$$\dot{m} C (T_o - T_i) = A_s h \left(T_w - \frac{T_i + T_o}{2} \right)$$

$$\dot{m} = \rho A_c U = 0.079 \text{ kg/s}$$

$$\therefore \quad 0.079 * 1006.3 (T_o - 15) = 2(0.13+0.07) * 10 * 29.84 \left(120 - \frac{15+T_o}{2} \right)$$

$T_o = 105°C \neq$ the first assumption

$\therefore$ the previous steps should be repeated at $T_o = 105$ untill the calculated T_o = the assumed value

Water flows inside tube having 25mm diameter and 4m length at the rate of 1 kg/s the wall temperature is maintained at constant temperature of 50°C and the inlet water temperature is 20°C. Determine the convection coefficient and the exit water temperature.

Note that: May you use the correlation equation for fluid flows through ducts as follows:

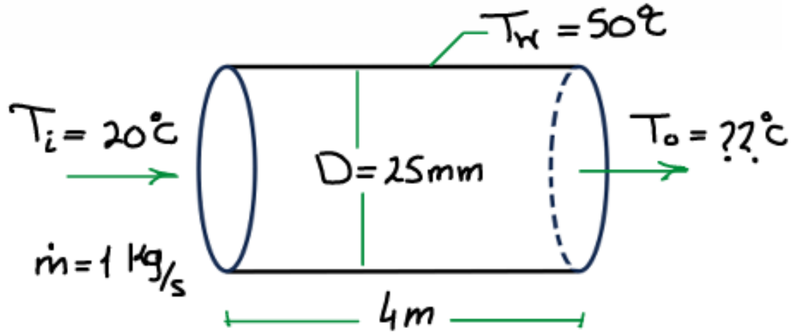
For <u>turbulent flow</u>, where Re > 2300

$Nu=0.023\ Re^{0.8}\ Pr^{n}$ **where n=0.3 for cooling and n=0.4 for heating**

For <u>Laminar flow</u>, where Re ≤ 2300

$Nu=1.86\ [Re.Pr]^{1/3}\ [D/L]^{1/3}\ [\mu/\mu_w]^{0.14}$

For both laminar and turbulent forced convection the properties are evaluated at mean bulk temperature, except μ_w at wall temperature.



assume $T_o = 34°C$

$$T_{mb} = \frac{T_o + T_i}{2} = \frac{34+20}{2} = 27°C = 300K$$

TABLE A.6 Thermophysical Properties of Saturated Water[a]

Temperature, T (K)	Pressure, p (bars)[b]	Specific Volume (m³/kg) $v_f \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg·K) $c_{p,f}$	$c_{p,g}$	Viscosity (N·s/m²) $\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m·K) $k_f \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr_f	Pr_g	Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta_f \cdot 10^6$ (K^{-1})	Temperature, T (K)
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1	300

$$\rho = \frac{1}{1.003 * 10^{-3}} = 997\ \frac{kg}{m^3},\quad C_p = 4179\ \frac{J}{kg.K},\quad \mu = 855 * 10^{-6}\ \frac{N.s}{m^2},\quad K = 0.613\ \frac{W}{m.k},\quad Pr = 5.83$$

$$\dot{m} = \rho A_c U = \rho \frac{\pi}{4} D^2 U = 997 * \frac{\pi}{4} * 0.025^2 U = 1 \longrightarrow U = 2.043 \ m/s$$

$$Re = \frac{\rho U D}{\mu} = \frac{997 * 2.043 * 0.025}{855 * 10^{-6}} = 59\,558 > 2300 \quad \text{turbulent flow}$$

$Nu=0.023\ Re^{0.8}\ Pr^{n}$ where n=0.3 for cooling and n=0.4 for heating

$$= 0.023 * 59\,558^{0.8} * 5.83^{0.4} = 307.57$$

$$h = \frac{K\ Nu}{D} = \frac{0.613 * 307.57}{0.025} = 7541.6 \ \frac{W}{m^2 K}$$

$$q = \dot{m} c (T_o - T_i) = A_s h (T_w - T_{mb})$$

$$= 1 * 4174 (T_{o_{calc}} - 20) = \pi * 0.025 * 4 * 7541.6 \left(50 - \frac{T_{o_{calc}} + 20}{2}\right)$$

$$\therefore T_{o_{calc}} = 33.2\ °C$$

$\therefore$ The assumed value of $T_o \neq$ the calculated value

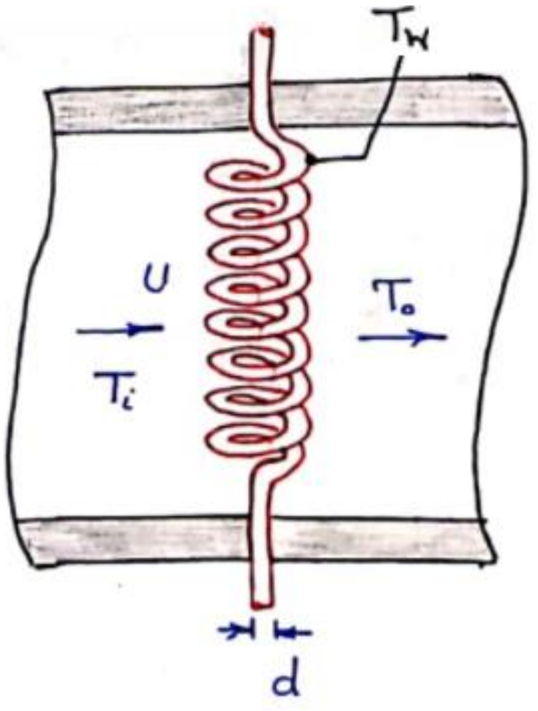
another trail should be done at $T_o = 33.2°C$ untill the calculated value

equate the assumed one, then calculate $q = \dot{m} c (T_o - T_i)$

c. For fluid flows over heater

$$Nu_d = 1.11\, b\, Re_d^{a}\, Pr^{0.31} \left[\frac{0.785\, T_{mb}}{T_w}\right]^{0.25}$$

Re_d	40 ~ 4000	4000 ~ 40000	40000 ~ 400000
a	0.466	0.618	0.805
b	0.015	0.0174	0.0204



$$Re_d = \frac{\rho U d}{\mu} \qquad Nu_d = \frac{h d}{k}$$

all properties evaluated at mean film temperature $T_{mf} = \dfrac{T_w + T_{mb}}{2}$

★ Solution steps :

- mean bulk temperature $T_{mb} = \frac{T_i + T_o}{2}$

- mean film temperature $T_{mf} = \frac{T_w + T_{mb}}{2}$

- Get ρ, c, μ, K and Pr from fluid tables at T_{mf}

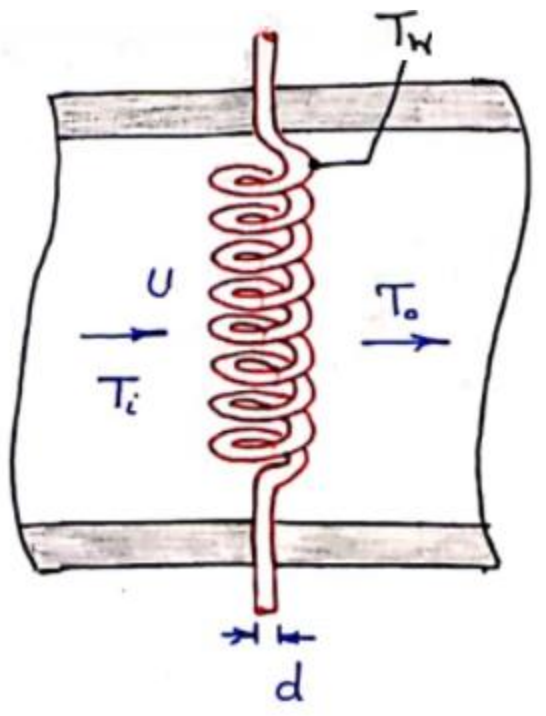
- $Re_d = \frac{\rho U d}{\mu}$ and determine the values of a and b

- $Nu_d = 1.11\ b\ Re_d^{a}\ Pr^{0.31} \left(\frac{0.785\ T_{mb}}{T_H} \right)^{0.25}$

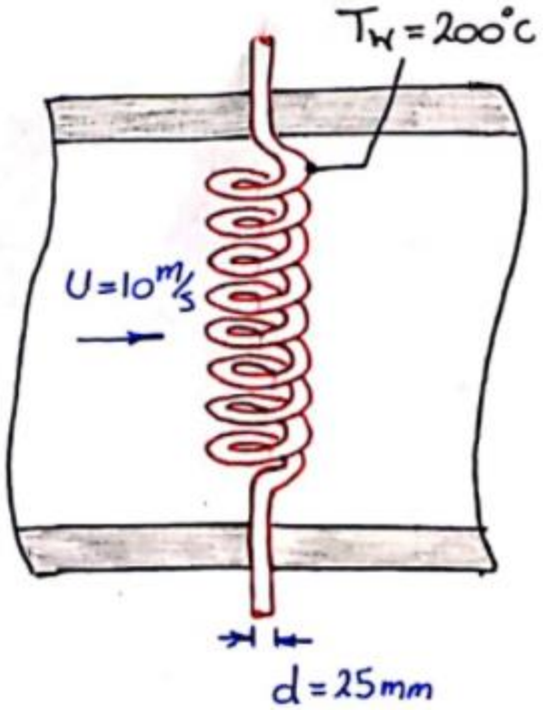
- Calculate the convection coefficient

$$Nu_d = \frac{hd}{K}$$

- $\dot{Q}_{heater} = A_w\ h\ (T_H - T_{mb})$



6-An electric wire having a diameter of 2.5mm and electric capacity of 2 kW, determine the wire length if the wire surface temperature not exceeds than 200°C when placed in air moves with velocity of 10 m/s and at average temperature of 54°C.



<u>Notes that</u>: You may use the following correlation equation for fluid flows over heating wire or heating rod having diameter of d as follows

$$Nu_d = 1.11 b Re_d^a Pr^{0.31} \left[\frac{0,785Tw}{Tm_b}\right]^{0.25}$$

where **a** and **b** are constant and can be determine as follows:

Re_d	40 ~ 4000	4000 ~ 40000	40000 ~ 400000
a	0.466	0.618	0.805
b	0.615	0.174	0.024

mean film temperature $T_{mf} = \frac{T_w + T_{mb}}{2} = 127°C = 400K$

$C = 1013.5\ \frac{J}{Kg.K}$ $\mu = 2.286 * 10^{-5}\ \frac{Kg}{m.s}$

$K = 0.03365\ \frac{W}{mK}$ $Pr = 0.688$

$\rho = 0.8824\ Kg/m^3$

TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m^3)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690

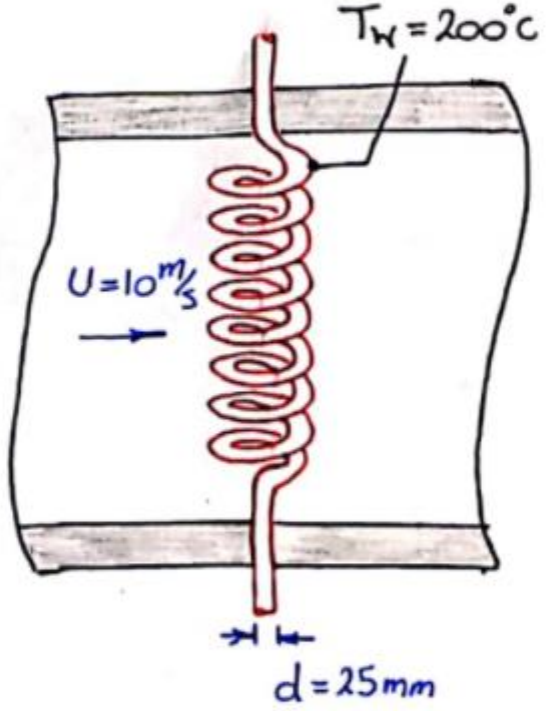
$$Re_d = \frac{\rho U d}{\mu} = \frac{0.8824 * 10 * 0.0025}{2.286 * 10^{-5}} = 965$$

Re_d	40 ~ 4000	4000 ~ 40000	40000 ~ 400000
a	0.466	0.618	0.805
b	0.615	0.174	0.024

$$\& \quad a = 0.466 \quad , \quad b = 0.615$$

$$Nu_d = 1.11\, b\, Re_d^{a}\, Pr^{0.31} \left[\frac{0.785\, T_w}{T_{mb}} \right]^{0.25}$$

$$= 1.11 * 0.615 * 965^{0.466} * 0.688^{0.31} \left[\frac{0.785 * 473}{54 + 273} \right]^{0.25} = 15.43$$



$$Nu_d = \frac{hd}{k} = \frac{0.0025\, h}{0.03365} \qquad h = 207.72\ \frac{W}{m^2 K}$$

$$\dot{q} = A_w\, h\, (T_w - T_{mb}) = \pi d L\, h\, (T_w - T_{mb})$$

$$= \pi * 0.0025\, L * 207.72\, (200 - 54)$$

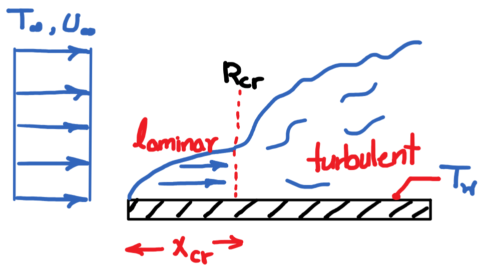
$$= 2000$$

$$L = 8.4\ m$$

Thank you

Forced convection part II

Flow over flat plate



Reynolds number at distance x $\quad Re_x = \dfrac{\rho U_\infty x}{\mu}$

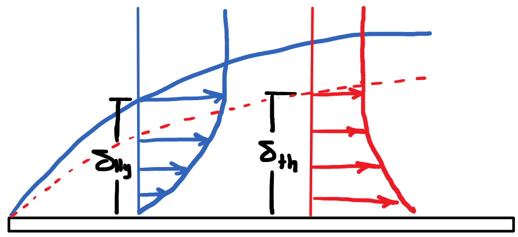
Critical Reynolds number $\quad R_{cr} = 5 * 10^5$

where $\quad R_{cr} = \dfrac{\rho U_\infty x_{cr}}{\mu} = 5 * 10^5$

$x < x_{cr}$ laminar region , $x > x_{cr}$ turbulent region

Hydraulic boundary layer thickness

$$\delta_{Hy} = \frac{4.64 x}{\sqrt{Re_x}}$$



Thermal boundary layer thickness

$$\delta_{th} = \frac{\delta_{Hy}}{Pr^{1/3}}$$

Empirical Nusselt equations

① local Nusselt number

- laminar $\quad Re_x \leq 5 * 10^5 \qquad Nu_x = 0.332\ Re_x^{\frac{1}{2}}\ Pr^{\frac{1}{3}}$
- Turbulent $\quad Re_x > 5 * 10^5 \qquad Nu_x = 0.029\ Re_x^{4/5}\ Pr^{\frac{1}{3}}$

② Average Nusselt number

- laminar $\quad Re_l \leq 5 * 10^5 \qquad Nu_l = 0.662\ Re_l^{\frac{1}{2}}\ Pr^{\frac{1}{3}}$
- Turbulent $\quad Re_l > 5 * 10^5 \qquad Nu_l = 0.037\ Re_l^{4/5}\ Pr^{\frac{1}{3}}$

All properties are evaluated at $\quad T_{mf} = \dfrac{T_\infty + T_w}{2}$

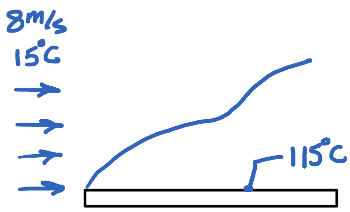
3-Air at temperature of 15°C flows with a velocity of 8 m/s at a flat plate which is maintained at a uniform temperature of 115°C. Calculate the thickness of the hydrodynamic and thermal boundary layers at 0.6m long from the leading edge also estimate the position of the point of transition to turbulent flow. Also calculate the av heat transfer coefficient at 0.6m from the leading edge of the plate then calculate the heat transfer rate from the plate to the air per unit width.

***Notes that**: May you use the correlation equation for fluid flows Over flat plate as follows:*

***For laminar region where** $Re_x \le 5x10^5$* $\quad Nu_{av} = 0.664 Re_x^{1/2}\ Pr^{1/3}$

***For Turbulent region where** $Re_x > 5x10^5$* $\quad Nu_{av} = 0.037 Re_x^{4/5} Pr^{1/3}$

Where $Nu_{av} = \frac{h.x}{k}$, $Re_x = \frac{\rho.U.x}{\mu}$ *and* $Pr = \frac{\mu.c_p}{k}$ *where all properties are evaluated at mean film temperature* $Tm_f = (T_w + T_\infty)/2$



$$Tm_f = \frac{T_w + T_\infty}{2}$$
$$= 65°C$$
$$= 238 K$$

From air table

$C_p = 1007.3$ J/kg.K

$\mu = 2.02 * 10^{-5}$ kg/ms

$k = 0.02913$ W/mK

$\rho = 1.046$ kg/m³

$Pr = 0.699$

Thickness of the hydrodynamic layer at 0.6 m

$$Re_x = \frac{\rho U_\infty x}{\mu} = 248554$$

$$\delta_{Hy} = \frac{4.64\, x}{\sqrt{Re_x}} = 0.00558 \text{ m}$$

Thermal boundary layer at 0.6 m

$$\delta_{th} = \frac{\delta_{Hy}}{Pr^{1/3}} = 0.00629 \text{ m}$$

The position of the transition

$$Re_{cr} = \frac{\rho U_\infty x_{cr}}{\mu} = 5 * 10^5 \rightarrow x_{cr} = 1.207 \text{ m}$$

Calculate h_{av} for $l = 0.6$ m

$$Re_{0.6} = 248554 < 5 * 10^5 \quad \text{laminar flow}$$

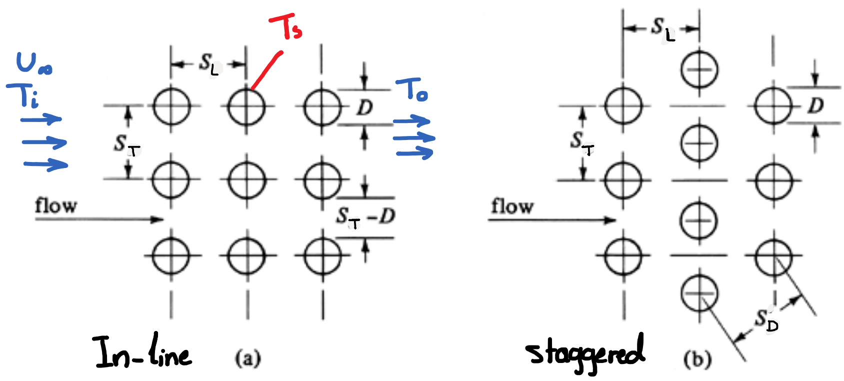
$$Nu_{av} = 0.664\, Re_l^{1/2}\, Pr^{1/3} = 293.79$$

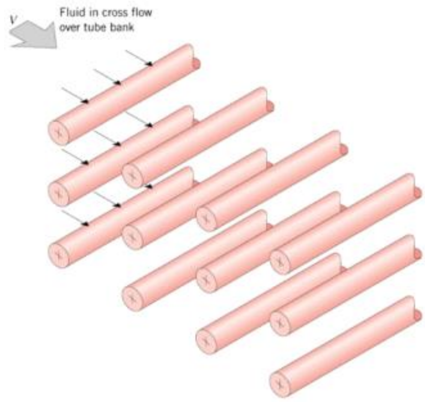
$$h_{av} = \frac{Nu_{av}\, K}{l} = 14.26 \text{ W/m}^2\text{K}$$

The heat transfer per unit width

$$Q = A_s h (T_w - T_\infty) = l\, \cancel{W}\, h (T_w - T_\infty) = 855.6 \text{ W/m.width}$$

Flow over bank of tubes





$$U_{max} = \frac{S_T}{S_T - D} U_\infty$$

if $S_T - D < 2(S_D - D)$ ∴ $U_{max} = \dfrac{S_T}{S_T - D} U_\infty$

if $S_T - D > 2(S_D - D)$ ∴ $U_{max} = \dfrac{S_T}{2(S_D - D)} U_\infty$

Empirical Nusselt equation

$$Nu = C_1 Re_{max}^{m} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4} \quad ; \text{ for } N_L \geq 20$$

$$Nu_{<20} = C_2 \, Nu \quad ; \text{ for } N_L < 20$$

where $Re_{max} = \dfrac{\rho U_{max} D}{\mu}$

$$Nu = \frac{h D}{k}$$

Conguration	$Re_{D,max}$	C_1	m
Aligned	10–10^2	0.80	0.40
Staggered	10–10^2	0.90	0.40
Aligned	10^2–10^3	Approximate as a single (isolated) cylinder	
Staggered	10^2–10^3		
Aligned ($S_T/S_L > 0.7$)[a]	10^3–2×10^5	0.27	0.63
Staggered ($S_T/S_L < 2$)	10^3–2×10^5	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered ($S_T/S_L > 2$)	10^3–2×10^5	0.40	0.60
Aligned	2×10^5–2×10^6	0.021	0.84
Staggered	2×10^5–2×10^6	0.022	0.84

[a]For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

All properties are evaluated at $T_{mb} = \dfrac{T_i + T_o}{2}$ exept Pr_s at T_s

Correction factor C_2 for $N_L < 20$

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

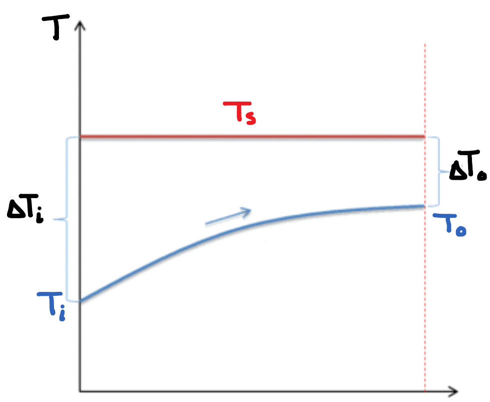
Heat transfer rate

$$q = A_s \, h \, \Delta T_{lm}$$

$A_s = N_T N_L \pi D \ell \longrightarrow$ طول الـ tube

N_L → عدد الـ tubes في الاتجاه الطولي

N_T → عدد الـ tubes في الاتجاه العرضي



$$\Delta T_{lm} = \frac{\Delta T_i - \Delta T_o}{\ln \frac{\Delta T_i}{\Delta T_o}}$$

from energy balance $A_s h \Delta T_{lm} = \dot{m} c_p (T_o - T_i)$

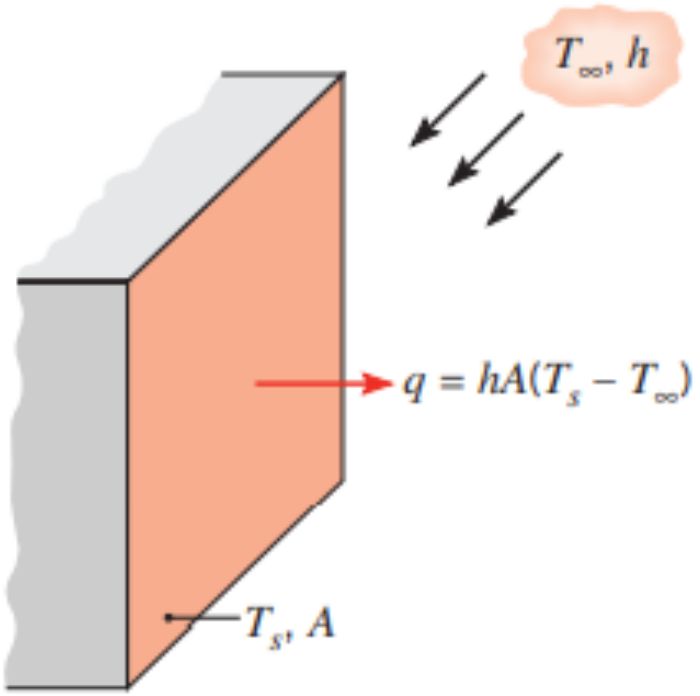
we got $T_o = T_s - (T_s - T_i) \, e^{\dfrac{-\pi D N_L N_T h}{\rho U_\infty N_T S_T c_p}}$

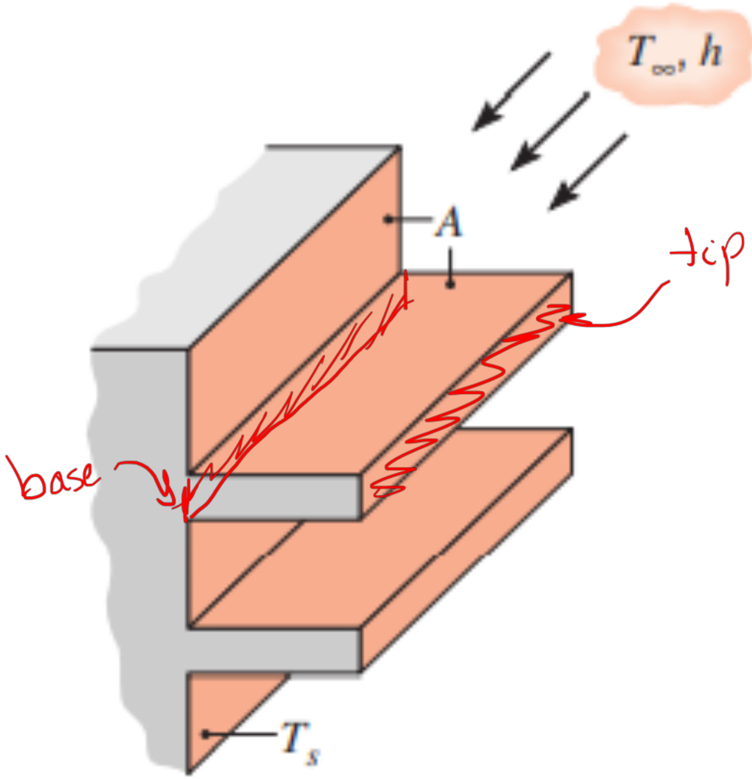
Heat transfer

<u>Conduction Heat Transfer from Extended Surfaces [Fins]</u>

Section No. 3

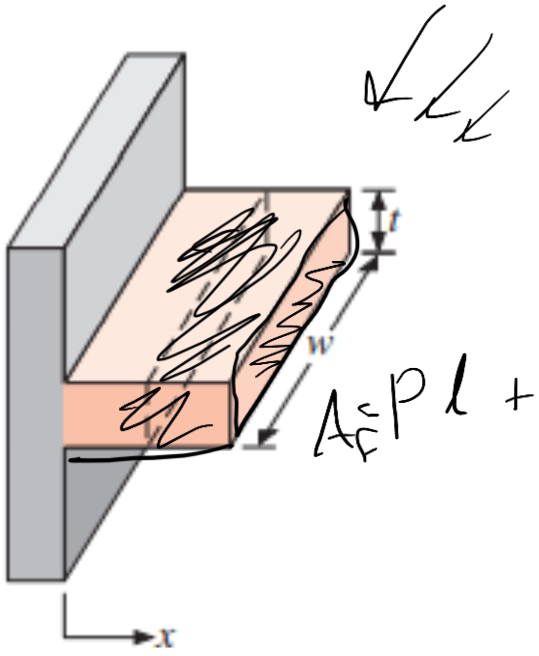
The extended surfaces "Fins"





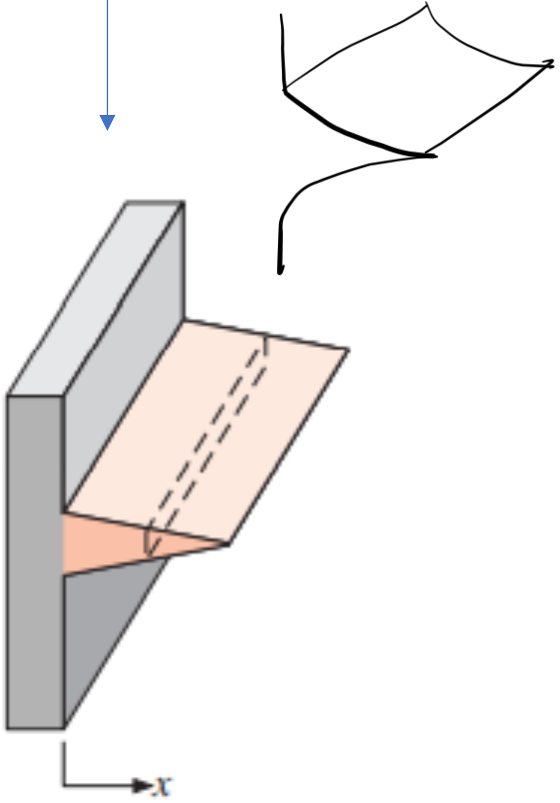
The extended surfaces are used to increase the effective area for convective heat transfer

Fin configurations.

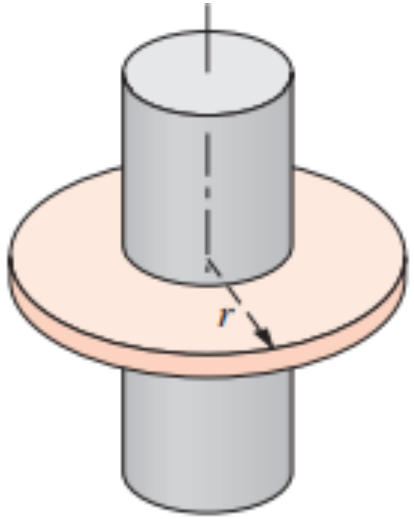


$A_f = P\,l + A_{c,t}$

Straight fin of uniform cross section.



Straight fin of nonuniform cross section.



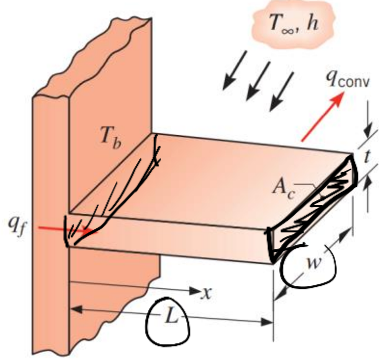
Annular fin.

Fin configurations

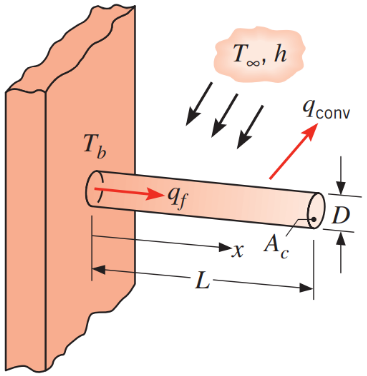
how to calc. q

- table
- Graphs η
- $\eta = \frac{\tanh mL_c}{mL_c}$

① Straight fin, uniform cross section area



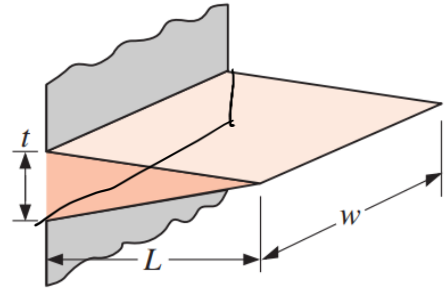
$$A_{c,b} = wt$$
$$A_{c,t} = wt$$
$$P = 2(w+t)$$
$$A_f = PL + A_{c,t}$$



$$A_{c,b} = A_{c,t} = \frac{\pi}{4} D^2$$
$$P = \pi D$$
$$A_f = PL + A_{c,t}$$

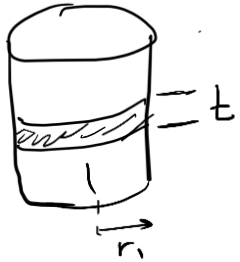
② Straight fin, Non-uniform cross section area

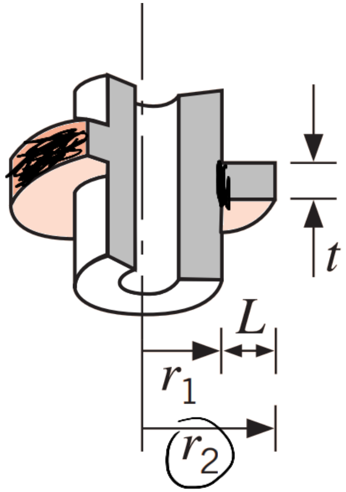
$$A_{c,b} = wt$$

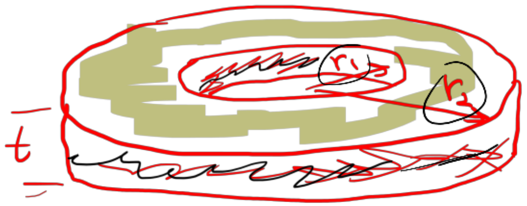


- graph

③ Annular fin

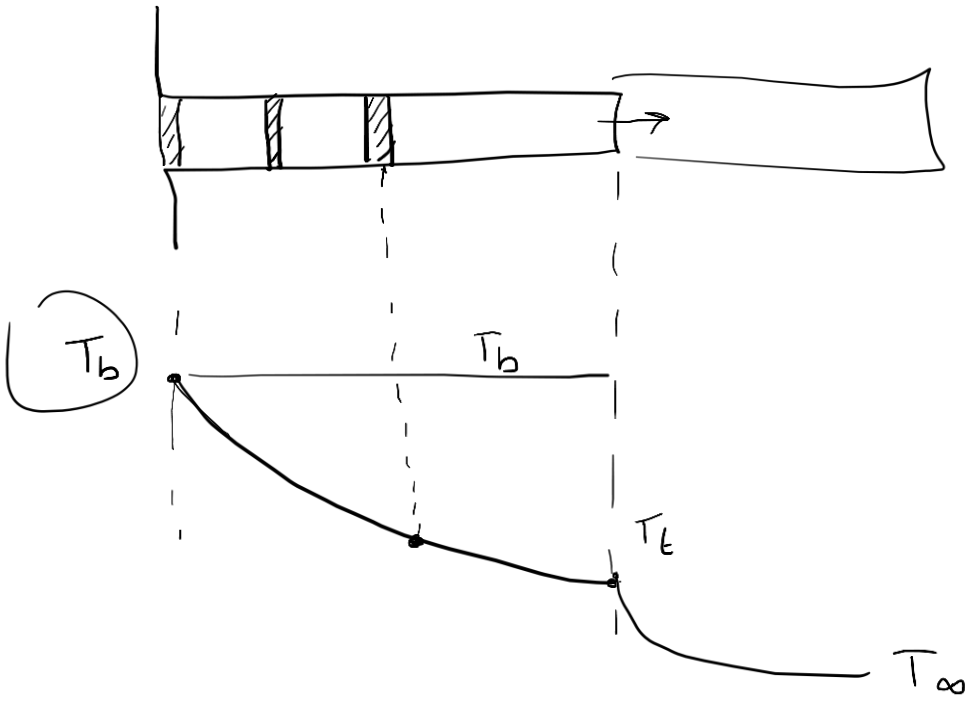






$$A_{c,b} = 2\pi r_1 t$$
$$A_{c,t} = 2\pi r_2 t$$
$$A_f = 2\pi (r_2^2 - r_1^2) + A_{c,t}$$

- graph



$$q = A h \Delta T$$

$$q_{f,max} = A_f h (T_b - T_\infty)$$

$$= A_f h \theta_b$$

$l_c = l + \begin{cases} \frac{t}{2} \\ \frac{D}{4} \end{cases}$

Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A active tip	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\frac{T_x - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B adiabatic tip	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\boxed{\frac{\cosh m(L-x)}{\cosh mL}}$	$\boxed{M \tanh mL}$
D very long fin	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	$\boxed{e^{-mx}}$	M

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$ $\quad (T_b - T_\infty)$

$$\boxed{m = \sqrt{\frac{hP}{kA_c}}}$$

$$\eta = \frac{q_F}{A_F h \theta_b}$$

$$\eta_{adiabatic} = \frac{\sqrt{hPkA_c}\,\cancel{\theta_b}\tanh ml}{Pl\,h\,\cancel{\theta_b}} = \sqrt{\frac{\cancel{h}\cancel{P}kA_c}{P^{\cancel{2}}h^{\cancel{2}}}}\frac{\tanh ml}{l} = \sqrt{\frac{kA_c}{Ph}}\frac{\tanh ml}{l} = \frac{\tanh ml}{ml}$$

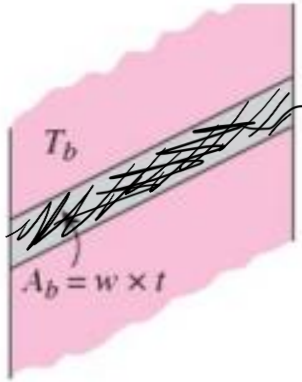
$$\eta = \frac{\tanh ml_c}{ml_c}$$

adiabatic $l_c = l$

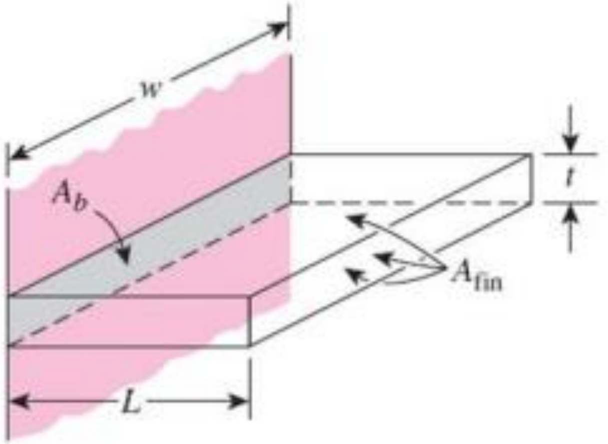
active $l_c = l + \frac{t}{2}$ / $\frac{D}{4}$

Fin Performance

$q_{r,c,b} = A_{c,b} h (T_b - T_\infty)$



(a) Surface without fins



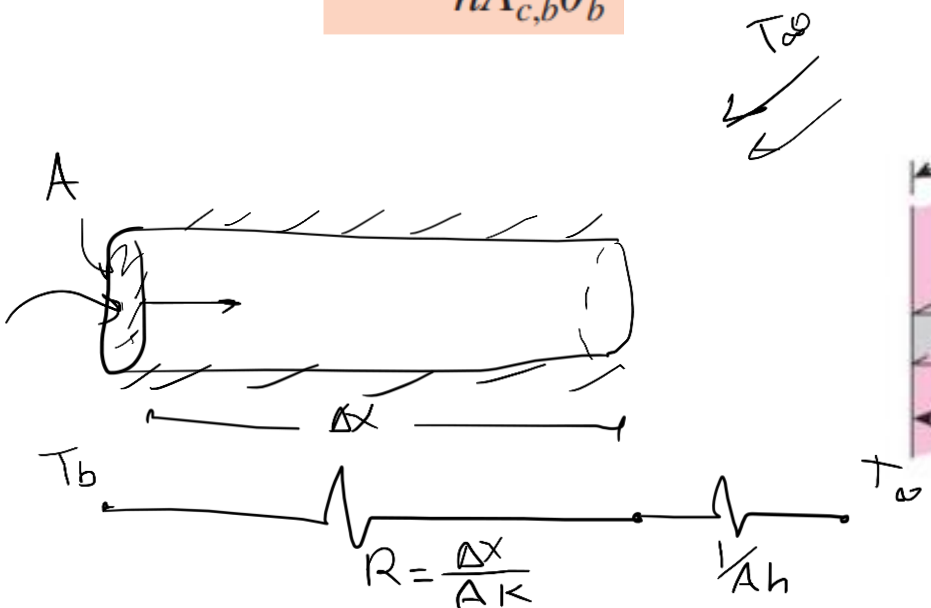
(b) Surface with a fin

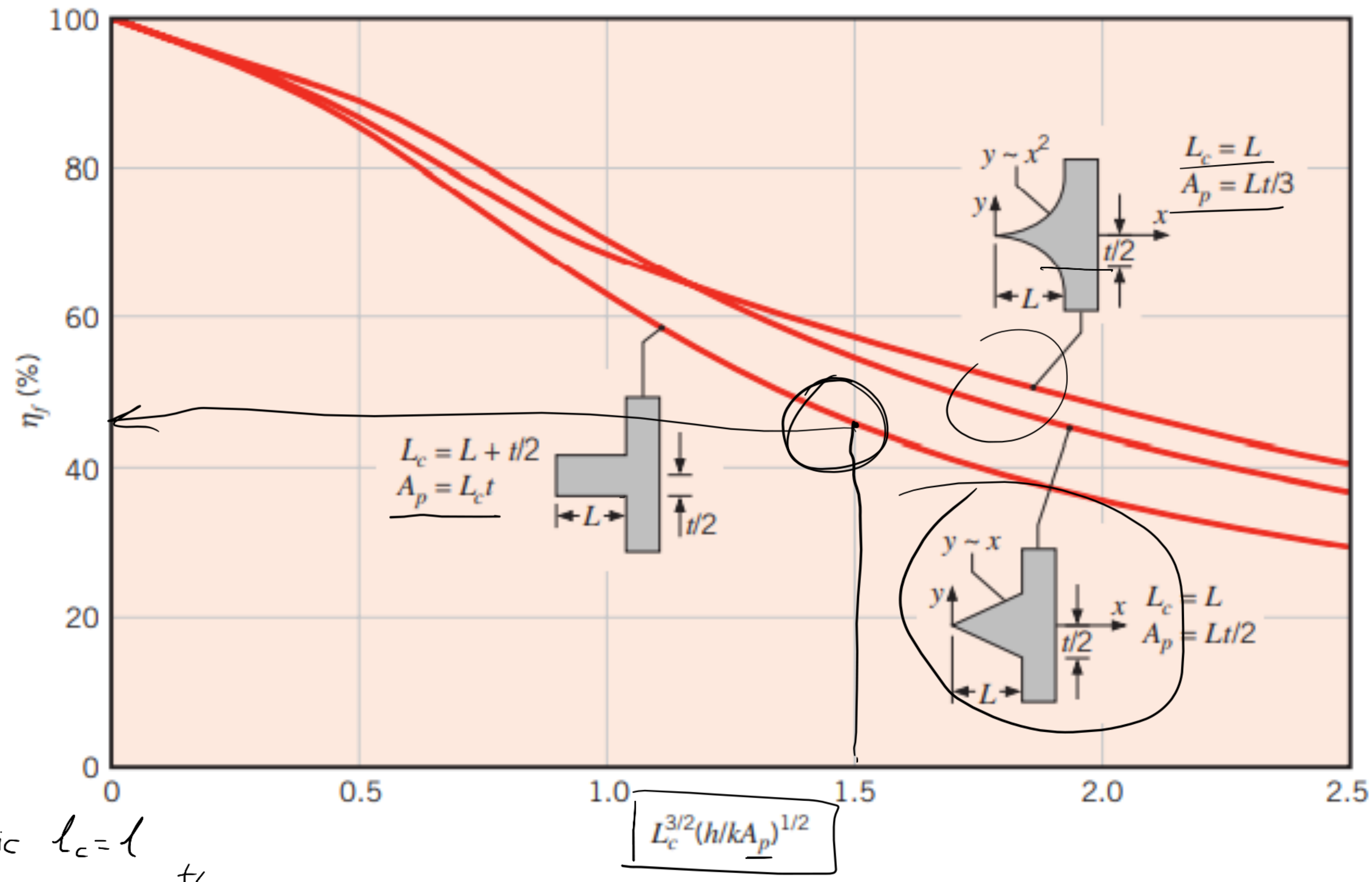
effectiveness ε_f

$$\varepsilon_f = \frac{q_f}{h A_{c,b} \theta_b}$$

Fin efficiency η_f

$$\eta_f \equiv \frac{q_f}{q_{max}} = \frac{q_f}{h A_f \theta_b} < 1$$

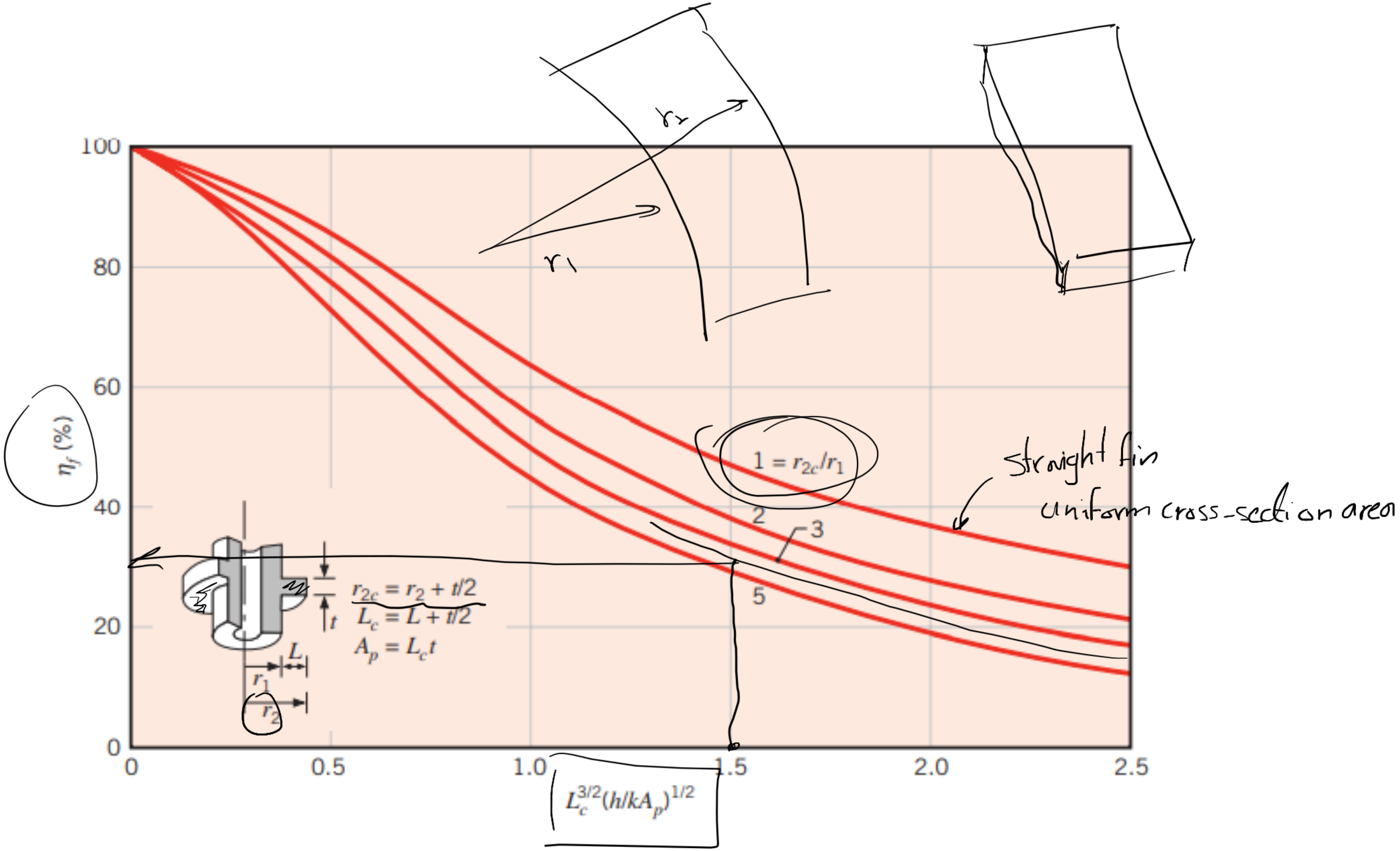




adiabatic $l_c = l$

active $l_c = l + \begin{cases} t/2 \\ D/4 \end{cases}$

Efficiency of straight fins (rectangular, triangular, and parabolic profiles).



Efficiency of annular fins of rectangular profile.

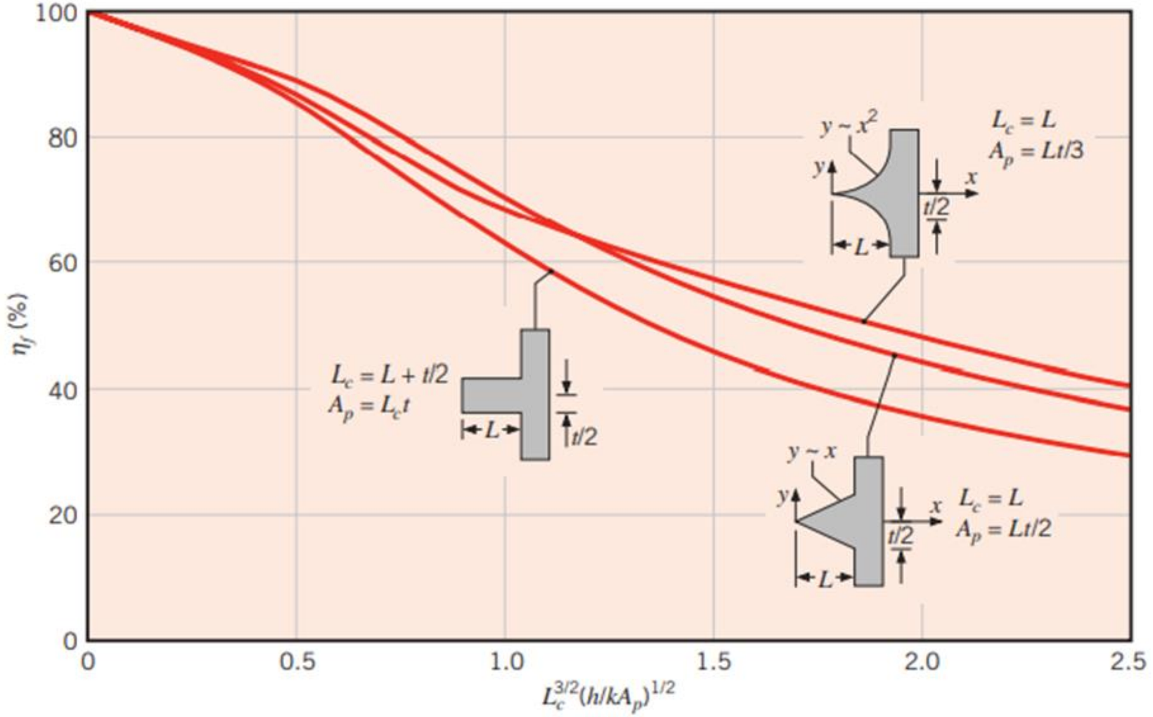
Temperature distribution and heat loss for fins of uniform cross section

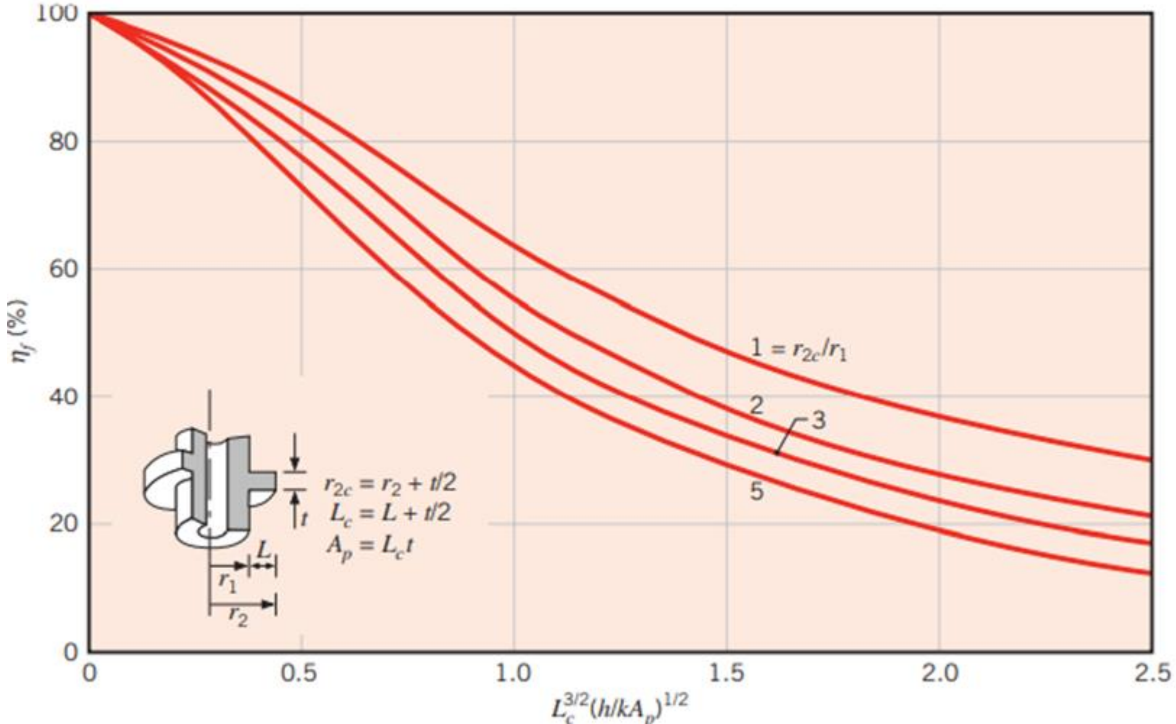
Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

$$q_f \equiv \eta_f\, q_{\max} = \eta_f\, h A_f \theta_b$$





1. A very long copper rod (k=372 W/m.K) 22mm in diameter has one end maintained at 90°C. The rod is exposed to a fluid whose temperature is 40°C and h=3.5W/m 2 .K. How much heat is lost by the rod? Also calculate the fin efficiency and effectiveness.

- Find q_f

Case D :

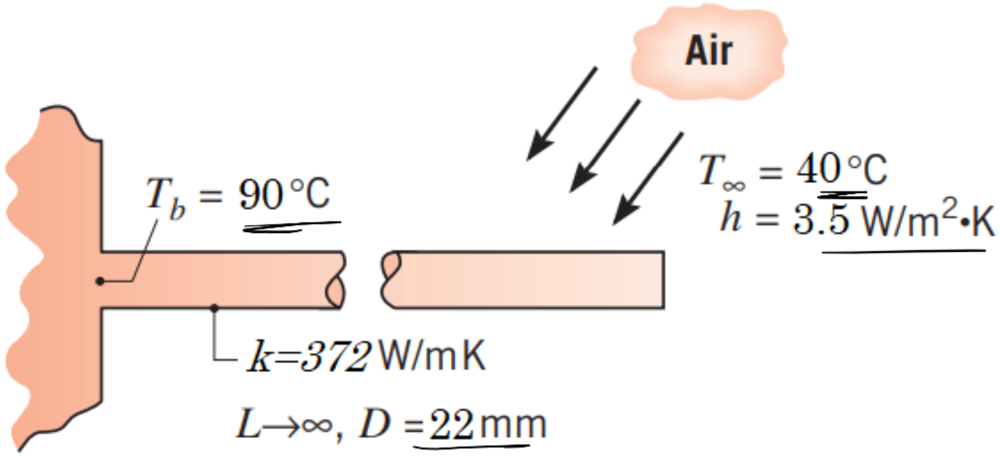
$$q_f = M = \sqrt{hPkA_c}\,(T_b - T_\infty)$$

$$P = \pi d = \pi * 0.022 = 0.0691\ m$$

$$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} * 0.022^2 = 0.00038\ m^2$$

$$q_f = \sqrt{3.5 * 0.0691 * 372 * 0.00038}\ (90-40)$$

$$= 9.245\ W$$



Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer:	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic:	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
D	Infinite fin ($L \to \infty$):	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

- Find η_f

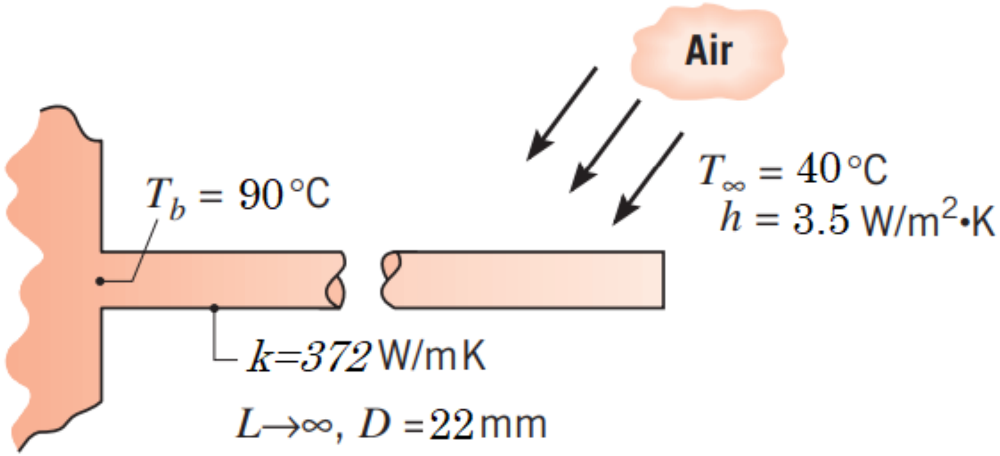
$$\eta_f = 0$$

- Find ε_f

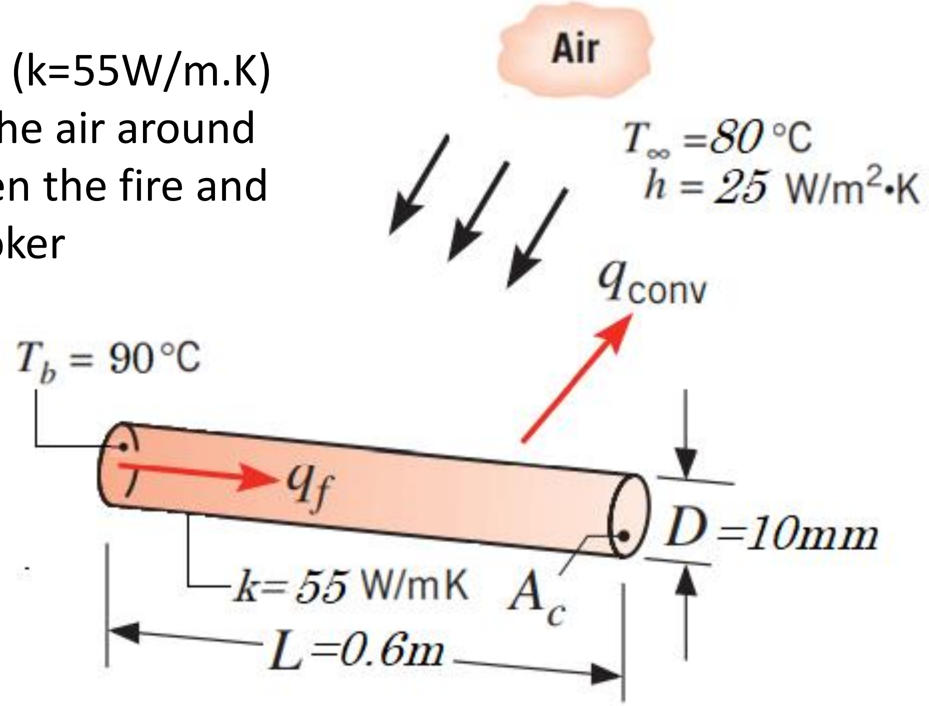
$$\varepsilon_f = \frac{q_f}{q_{w/o\,f}} = \frac{q_f}{A_{c,b}\, h\,(T_b - T_\infty)}$$

$$= \frac{9.245}{\frac{\pi}{4} * 0.022^2 * 3.5\,(90-40)}$$

$$= 139$$



4. One end of a circular poker is places in a fire. The poker is made of steel (k=55W/m.K) and it has a diameter of 10mm. The end of the poker in the fire is 350°C, the air around the poker is at 80°C with h=25 W/m 2 .K. If the length of the poker between the fire and the handle is 0.6m long, calculate the temperature of the handle of the poker



- Calculate the temperature of the handle of the poker

Case A :

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$P = \pi d = \pi * 0.01 = 0.0314\ m$$

$$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} * 0.01^2 = 0.0000785\ m^2$$

$$m = \sqrt{\frac{25 * 0.0314}{55 * 0.0000785}} = 13.484\ m^{-1}$$

Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer:	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic:	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
D	Infinite fin ($L \to \infty$):	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

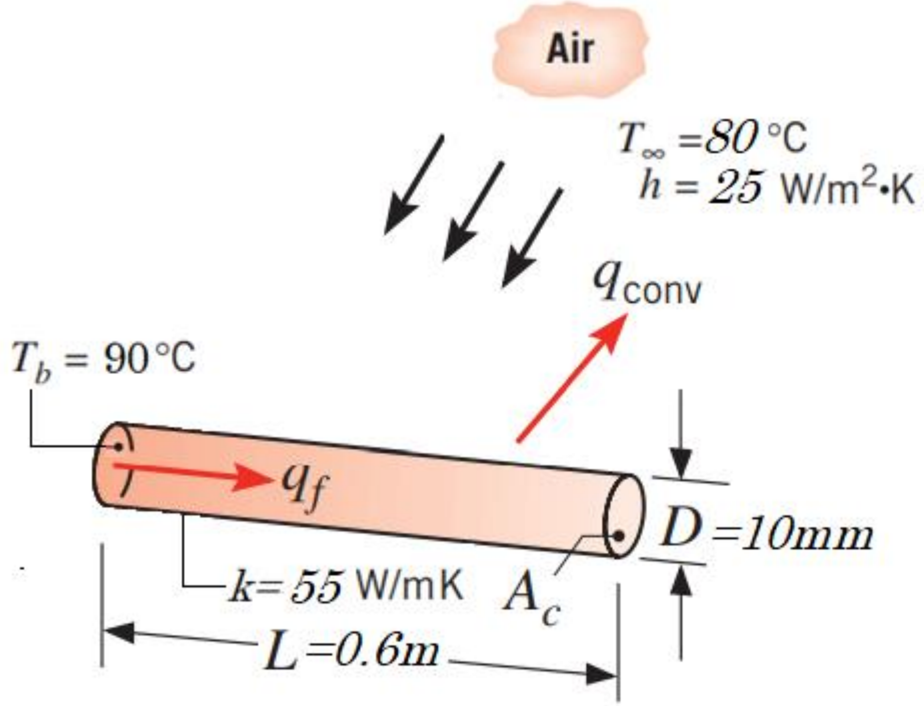
$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

T_t at $x = L$

$$\frac{T_t - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(13.484 * 0.6) + \left(\frac{25}{13.484 * 55}\right)\sinh(13.484 * 0.6)}$$

$$= 0.000593$$

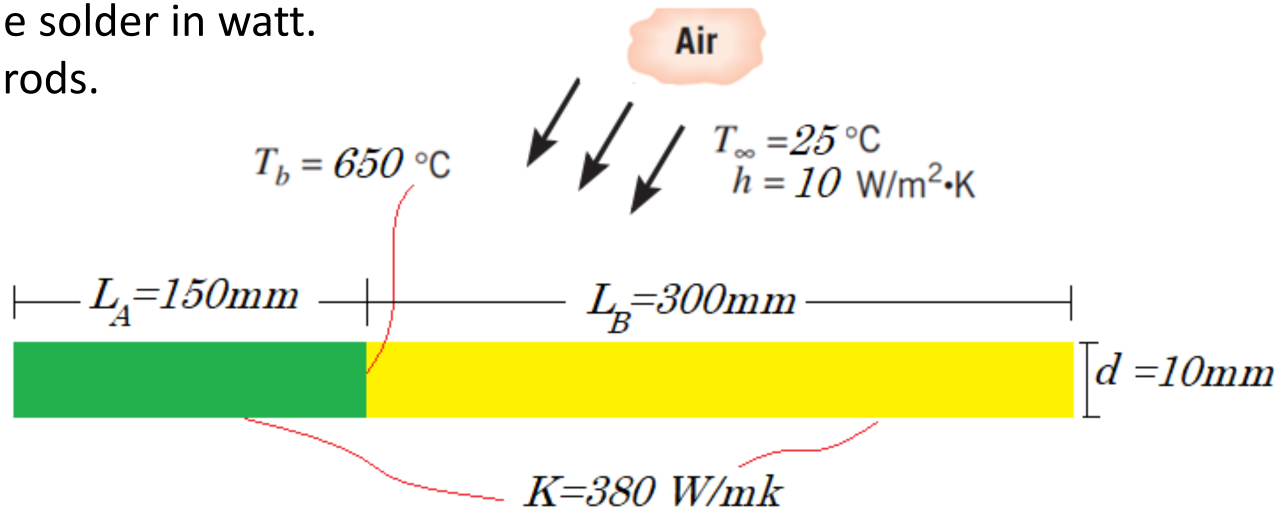
$$T_t = 80.16°C$$



3. Two copper rods A & B (380W/m.K) having 10mm diameter and length of L A =150mm and L B =300mm The rods are to be soldered together end to end with soldering material having melting point at 650°C. The two rods are exposed to ambient temperature T ∞ =25°C and h=10 W/m 2 .K. Neglecting the heat transfer from rod tips,

Calculate: i-the minimum power input needed to affect the solder in watt.

ii-The temperature at the free ends of the both rods.



- The minimum power input needed

$\boxed{\text{Rod A:}}$ "Case B" $q_f = M \tanh mL$

$$M = \sqrt{hpKA_c}\,(T_b - T_\infty) \qquad m = \sqrt{\frac{hP}{KA_c}}$$

$$p = \pi d = \pi * 0.01 = 0.0314 \text{ m}$$

$$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} * 0.01^2 = 0.0000785 \text{ m}^2$$

$$M = \sqrt{10 * 0.0314 * 380 * 0.0000785}\,(650 - 25)$$

$$= 60.488 \text{ W}$$

$$m = \sqrt{\frac{10 * 0.0314}{380 * 0.0000785}} = 3.244 \text{ m}^{-1}$$

Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer:	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic:	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
D	Infinite fin ($L \to \infty$):	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

$$q_{f_A} = 60.488 \tanh(3.244 * 0.15) = 27.3\ W$$

$\boxed{\text{Rod B :}}$ "Case B" $q_f = M \tanh mL$

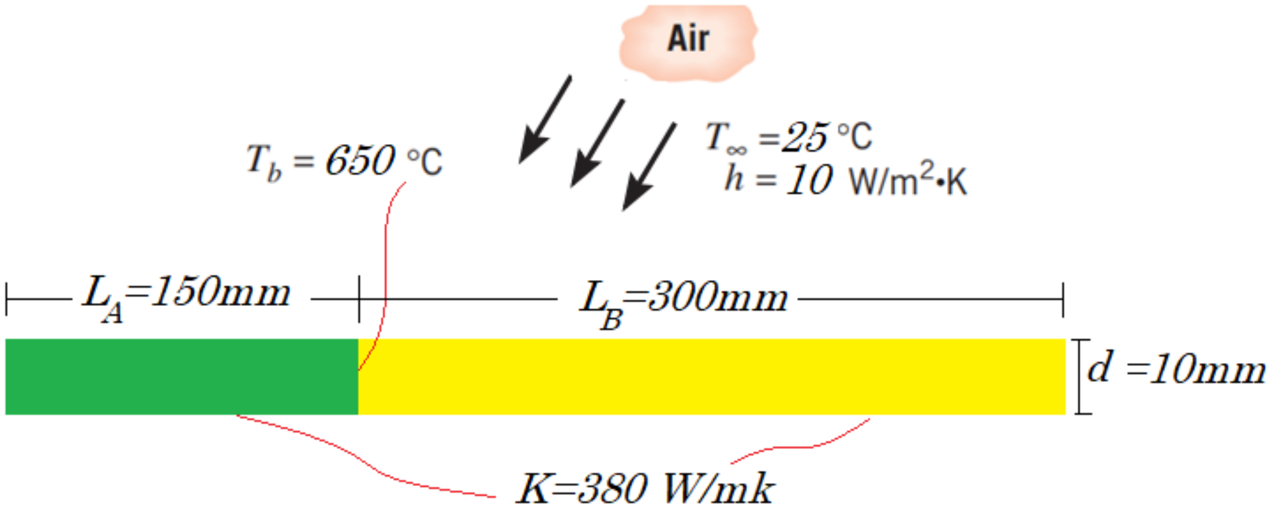
$$M = \sqrt{hpKA_c}\,(T_b - T_\infty) \qquad m = \sqrt{\frac{hP}{KA_c}}$$

$$q_{f_B} = 60.488 \tanh(3.244 * 0.3) = 45.37\ W$$

minimum power input $= q_{f_A} + q_{f_B}$

$= 27.3 + 45.37$

$= 72.67\ W$



- The temperature at the free ends

$\boxed{\text{Rod A :}}$ "Case B" $\dfrac{\theta}{\theta_b} = \dfrac{\cosh m(L-x)}{\cosh mL}$

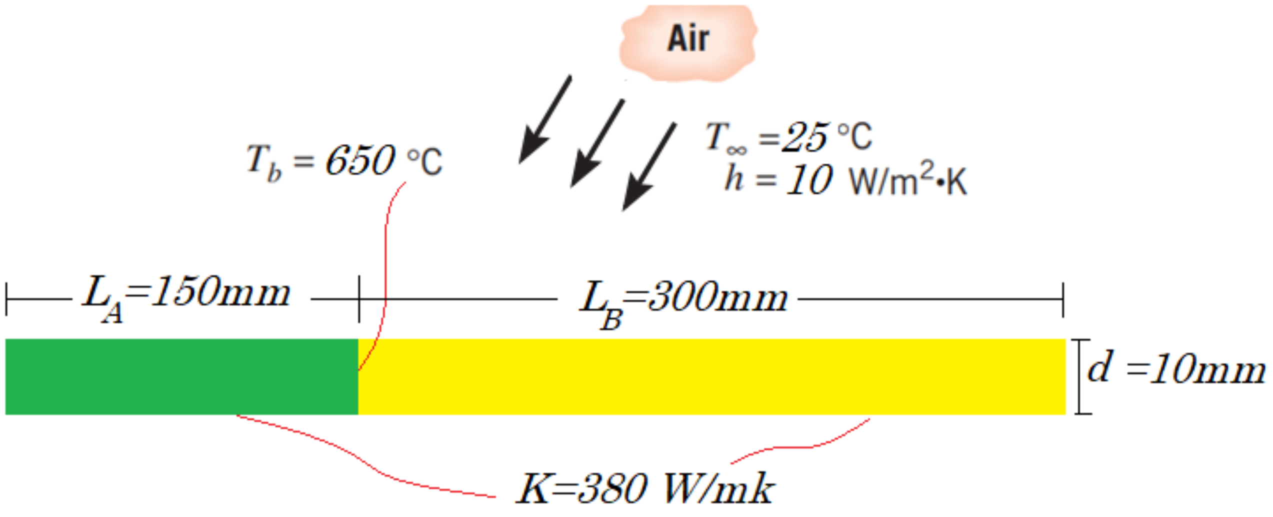
$$\frac{T_{t,A} - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(3.244 * 0.15)}$$

$$T_{t,A} = 582.7\ ^\circ C$$

$\boxed{\text{Rod B :}}$ "Case B" $\dfrac{\theta}{\theta_b} = \dfrac{\cosh m(L-x)}{\cosh mL}$

$$\frac{T_{t,B} - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(3.244 * 0.3)}$$

$$T_{t,B} = 438.3\ ^\circ C$$



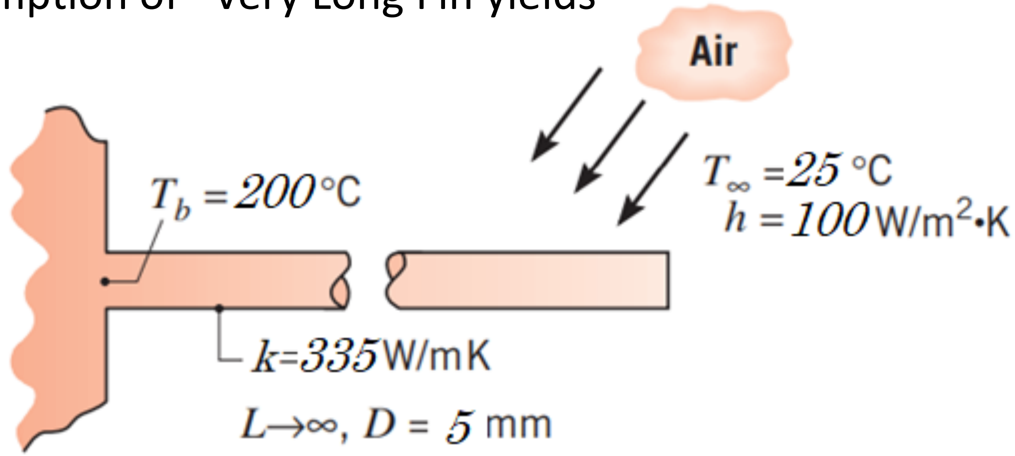
Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer:	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic:	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
D	Infinite fin ($L \rightarrow \infty$):	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

5. A very long pin fin of 5mm diameter has a base temperature of $T_b = 200\ ^oC$. The pin surrounding surface is exposed to air at $T_\infty = 25\ ^oC$ with $h = 100\ W/m^2.K$. Calculate the heat loss through the fin and the fin effectiveness. Also estimate how long the fin must have to be making assumption of "Very Long Fin yields to accurate estimation for heat loss
calculation. Take for fin material $k = 335\ W/m.K$.



- Calculate the heat loss through the fin

Case D $\quad q_f = M = \sqrt{hpkA_c}\,(T_b - T_\infty)$

$p = \pi d = \pi * 0.005 = 0.0157\ m$

$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} * 0.005^2 = 0.0000196\ m^2$

$q_f = \sqrt{100 * 0.0157 * 335 * 0.0000196}\ (200 - 25)$

$= 17.77\ W$

Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer:	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic:	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
D	Infinite fin ($L \to \infty$):	e^{-mx}	M

$\theta \equiv T - T_\infty \qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty \qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

- The fin effectiveness

$$\varepsilon_f = \frac{q_f}{q_{w/o\,f}}$$

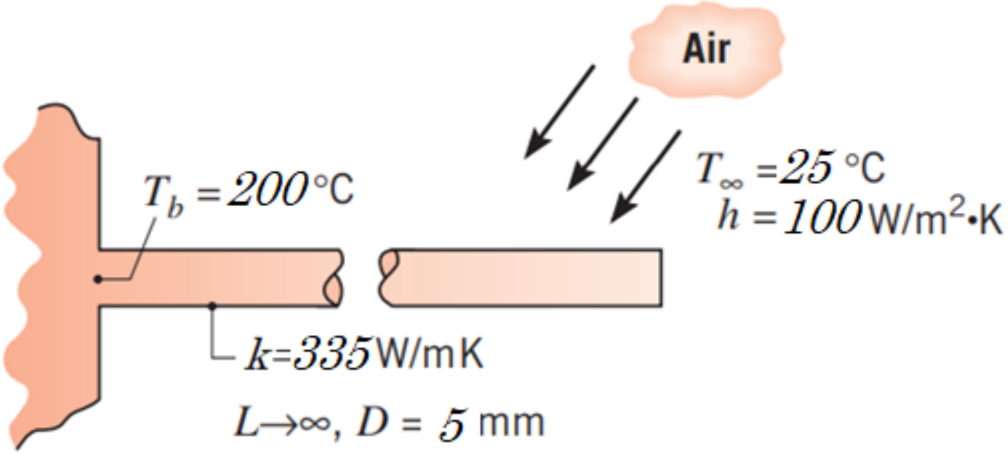
$$q_{w/o\,f} = A_{c,b}\, h\,(T_b - T_\infty)$$

$$= \frac{\pi}{4} * 0.005^2 * 100\,(200 - 25)$$

$$= 0.3436 \text{ W}$$

$$\varepsilon_f = \frac{q_f}{q_{w/o\,f}}$$

$$= \frac{17.77}{0.3436} = 51.7$$



- Estimate how long the fin must have to be making assumption of very long fin yields to accurate estimation for heat loss

$$q_{f,\text{adiabatic tip}} = 0.99999\ q_{f,\text{very long fin}}$$

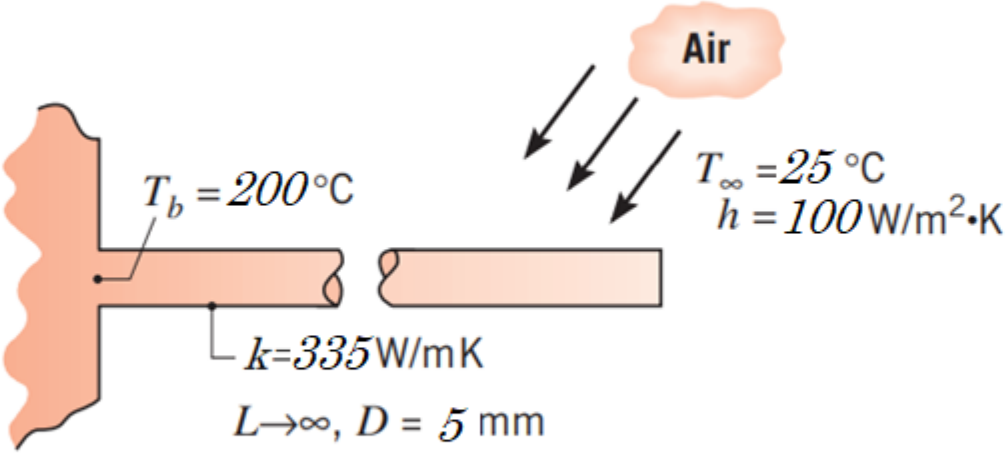
$$M \tanh mL_{eff} = 0.99999\ M$$

$$mL_{eff} = 5$$

$$L_{eff} = \frac{5}{m}$$

$$m = \sqrt{\frac{hP}{KA_c}} = \sqrt{\frac{100 * 0.0157}{335 * 0.0000196}} = 15.46\ m^{-1}$$

$$L_{eff} = \frac{5}{15.46} = 0.323\,m$$



Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
A	Convection heat transfer:	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic:	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
D	Infinite fin ($L \to \infty$):	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

7. In an arrangement for measuring the thermal conductivity of solid material, a very long rod having diameter of 25mm is introduced inside a furnace such that half of it was inside the furnace, while the other half extended horizontally in ambient air at T ∞ =25°C with h=10W/m 2 .K. By measuring the temperature at two points on the rod, the temperature was found to be 150°C and 95°C, respectively while the two points are at a distance of 75mm from each other. Find the thermal conductivity of the rod material.

- Find the thermal conductivity of the rod material

Case D :
$$\frac{\theta}{\theta_b} = e^{-mx}$$

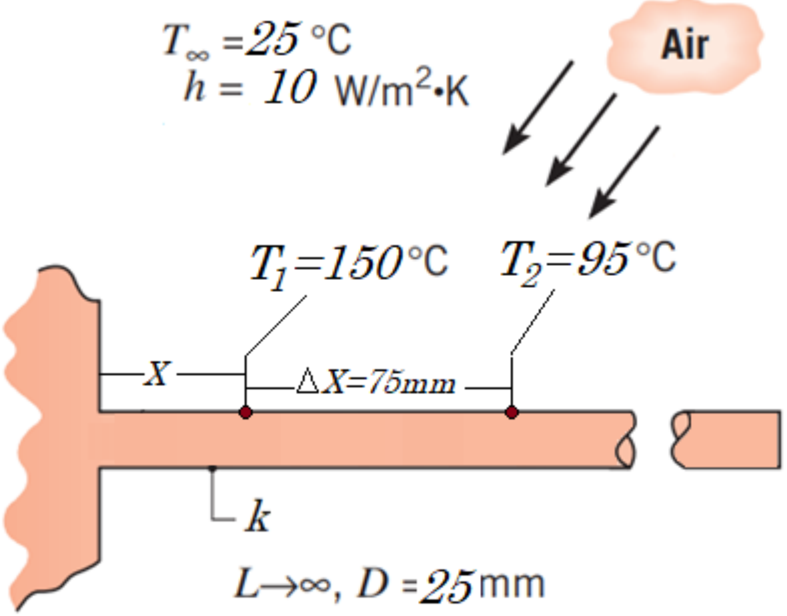
$$m = \sqrt{\frac{hP}{KA_c}}$$

$$P = \pi d = \pi * 0.025 = 0.0785 \text{ m}$$

$$A_c = \frac{\pi}{4} d^2 = \frac{\pi}{4} * 0.025^2 = 0.00049 \text{ m}^2$$

$$m = \sqrt{\frac{10 * 0.0785}{0.00049 K}}$$

$$= 40.0255 \ K^{-0.5}$$



Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
D	Infinite fin ($L \rightarrow \infty$):	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

For node ①
$$\frac{150-25}{\theta_b} = e^{-mx}$$

For node ②
$$\frac{95-25}{\theta_b} = e^{-m(x+0.075)}$$

From node ① and ②
$$\frac{125}{e^{-mx}} = \frac{70}{e^{-m(x+0.075)}}$$

$$\frac{125}{70} = \frac{e^{-mx}}{e^{-m(x+0.075)}} = e^{-mx+m(x+0.075)}$$

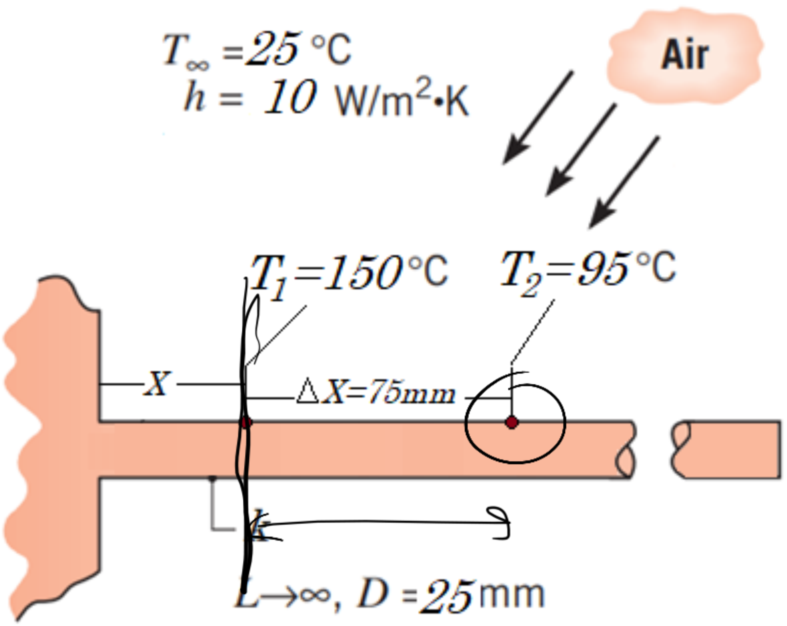
$$\frac{125}{70} = e^{0.075m}$$

$$\ln\frac{125}{70} = 0.075\,m = 0.075 * 40.0255\,K^{-0.5}$$

$$K = 26.8\ \frac{W}{mK}$$

$$\frac{95-25}{150-25} = e^{-m*0.075}$$

$m = \checkmark \longrightarrow K = \checkmark$



Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q
D	Infinite fin ($L \rightarrow \infty$):	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

Thank you

Heat transfer

Conduction Heat Transfer from Extended Surfaces [Fins]

Section No. 4

Single fin

① Straight fin

Case A:

$$q_f = M \frac{\sinh mL + (h/mK)\cosh mL}{\cosh mL + (h/mK)\sinh mL} \qquad q_f = \eta_f A_f h \theta_b \qquad \eta_f = \frac{\tanh mL_c}{mL_c}$$

$$l_c = l + \frac{A_c}{P}$$

→ thin rectangular $\frac{t}{2}$

→ Circle $\frac{D}{4}$

Case B:

$$q_f = M \tanh mL \qquad q_f = \eta_f A_f h \theta_b$$

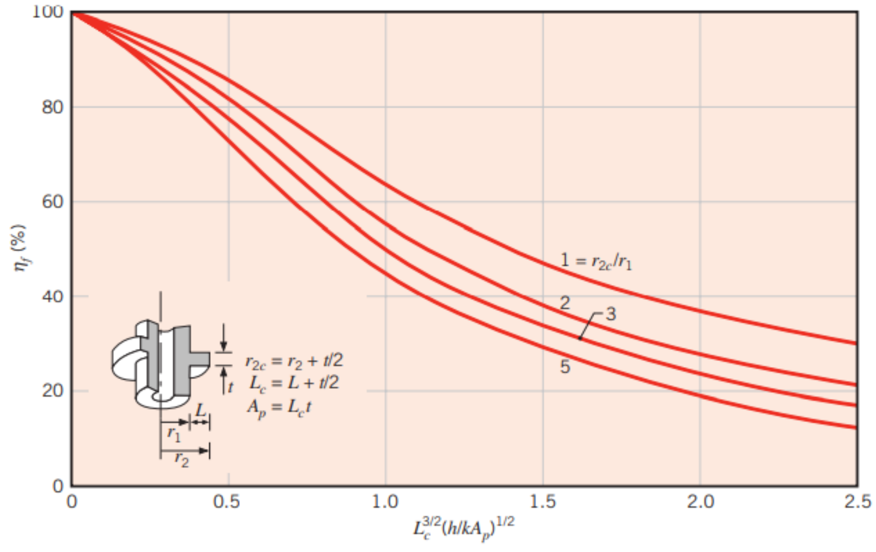
$$\eta_f = \frac{q_f}{q_{max}} = \frac{M \tanh mL}{A_f h \theta_b} = \frac{\sqrt{hPKA_c}\,\theta_b \tanh mL}{PLh\theta_b} = \sqrt{\frac{hPKA_c}{P^2h^2}}\,\frac{\tanh mL}{L} = \frac{\tanh mL}{mL}$$

Case D:

$$q_f = M$$

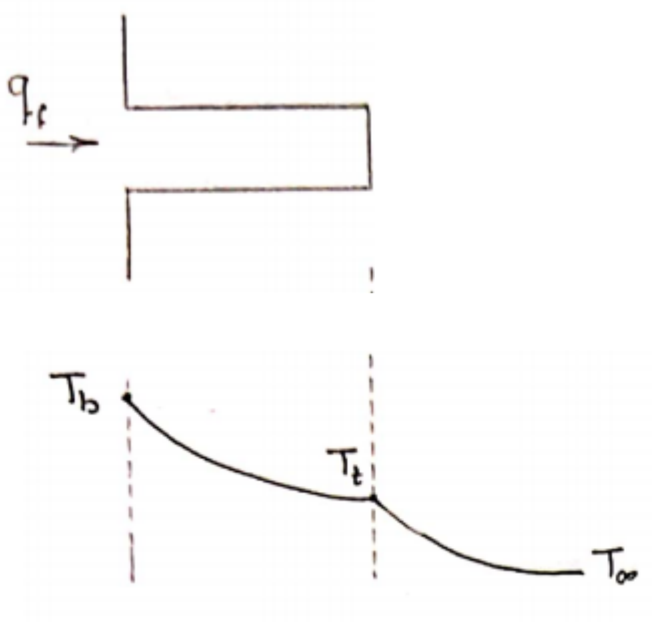
② Annular fin

$$q_f = \eta_f A_f h \theta_b$$



Efficiency of annular fins of rectangular profile.

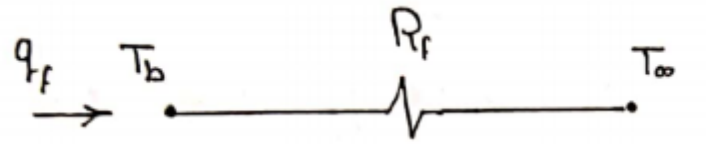
Fin Resistance



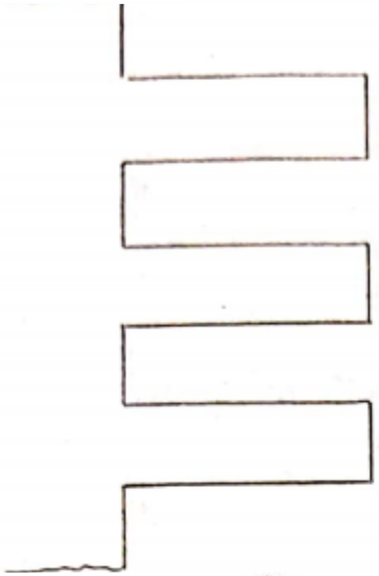
$$q_f = \frac{T_b - T_\infty}{R_f}$$

$$= \eta_f A_f h (T_b - T_\infty)$$

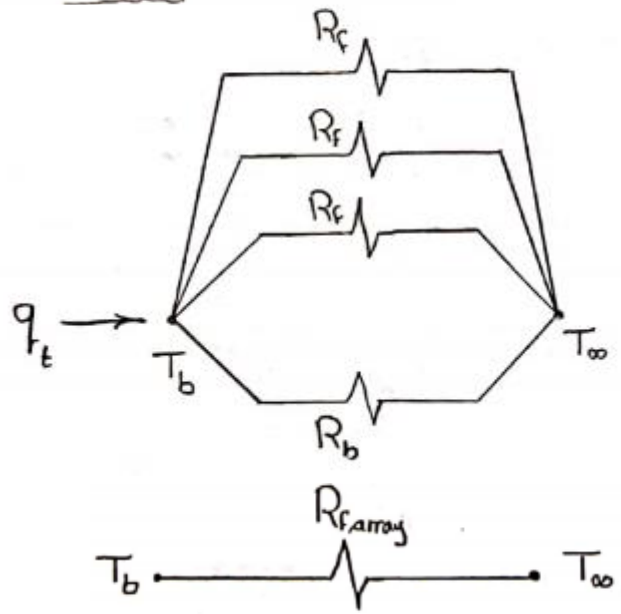
$$R_f = \frac{1}{\eta_f A_f h}$$



Fins array



$$A_t = A_{base} + N_f A_f$$



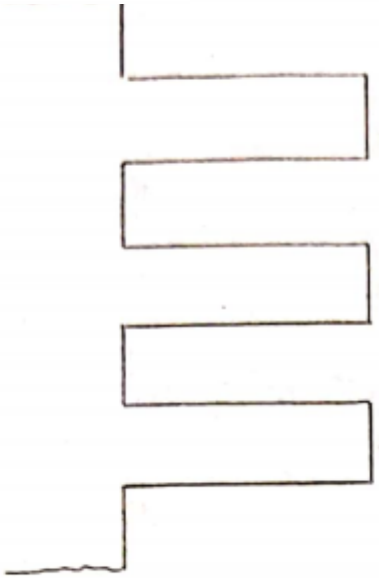
Total rate of heat transfer from fins array

$$
\begin{aligned}
q_t &= q_{base} + N_f q_f \\
&= A_b h \Theta_b + N_f \eta_f A_f h \Theta_b \\
&= h \Theta_b \left[A_b + N_f \eta_f A_f \right] \\
&= A_t h \Theta_b \left[\frac{A_b}{A_t} + \frac{N_f A_f}{A_t} \eta_f \right] \\
&= A_t h \Theta_b \left[\frac{A_t - N_f A_f}{A_t} + \frac{N_f A_f}{A_t} \eta_f \right] \\
&= A_t h \Theta_b \left[1 + \frac{N_f A_f}{A_t} (\eta_f - 1) \right]
\end{aligned}
$$

Total efficiency

$$\eta_t = \frac{q_t}{q_{t,max}} = \frac{q_t}{A_t h \Theta_b} = 1 + \frac{N_f A_f}{A_t} (\eta_f - 1)$$

Fins array



$$A_t = A_{base} + N_f A_f$$

Total efficiency

$$\boxed{\eta_t = 1 + \frac{N_f A_f}{A_t}(\eta_f - 1)}$$

$$q_t = \eta_t A_t h \Theta_b = \frac{\Theta_b}{R_{f,array}}$$

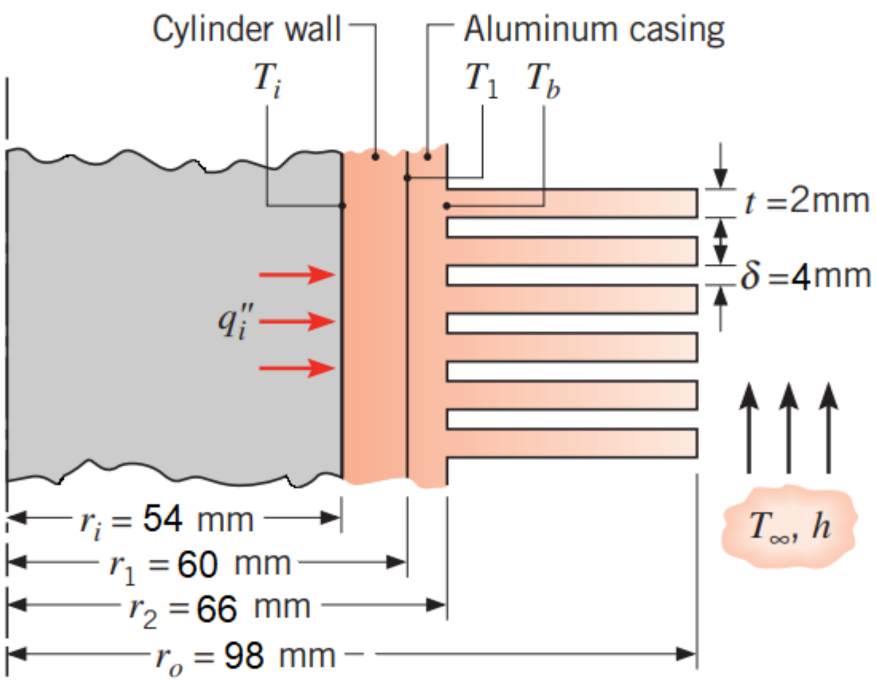
$$\boxed{R_{f,array} = \frac{1}{\eta_t A_t h}}$$

9. If it is proposed to air-cool the cylinder wall of a combustion chamber by adding an aluminium casing with 30 annular fins of k= 215 W/m.K, to the cylinder wall of k=50W/m.K, as shown in the figure. The cold air is at 45°C with h=150 W/m^2.K and the heat transfer rate from the inner cylinder wall and the casing is Q°= 7.5 kW. If the contact resistance between the cylinder wall and the casing is 10^{-4} m^2.k/W, draw the temperature distribution and the thermal resistance for the configuration and ***calculate:***

i- The fin efficiency η_f and total efficiency η_t.

ii- The temperature of the inner and outer cylinder wall (T_i & T_o) and the fin base temperature (T_b).

iii- The fins effectiveness, ε (Fin Heat Ratio **FHR*)***.



i. Calculate η_f and η_t

$$L_c = L + \frac{t}{2} = 0.032 + \frac{0.002}{2} = 0.033 \text{ m}$$

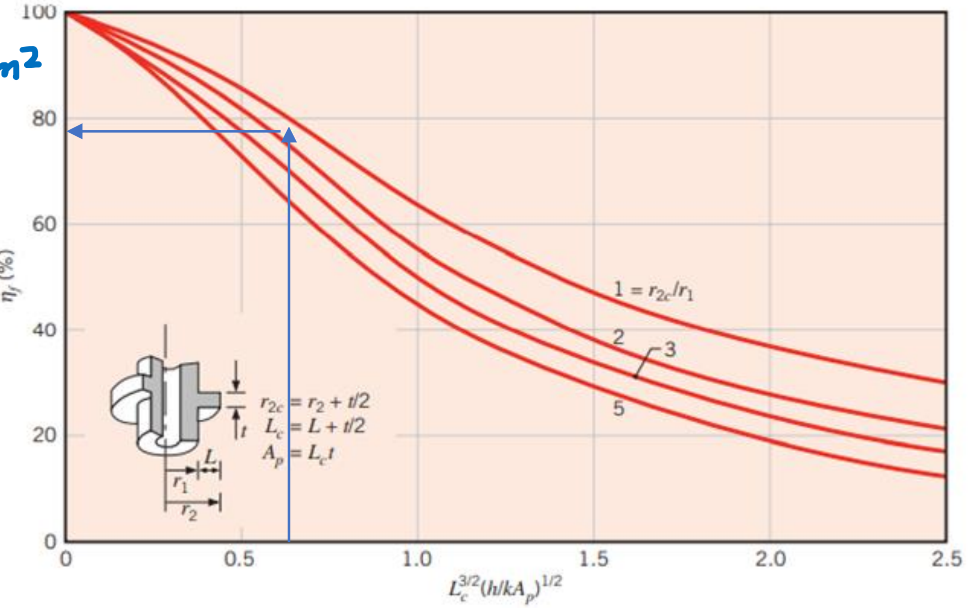
$$A_p = L_c t = 0.033 * 0.002 = 6.6*10^{-5} \text{ m}^2$$

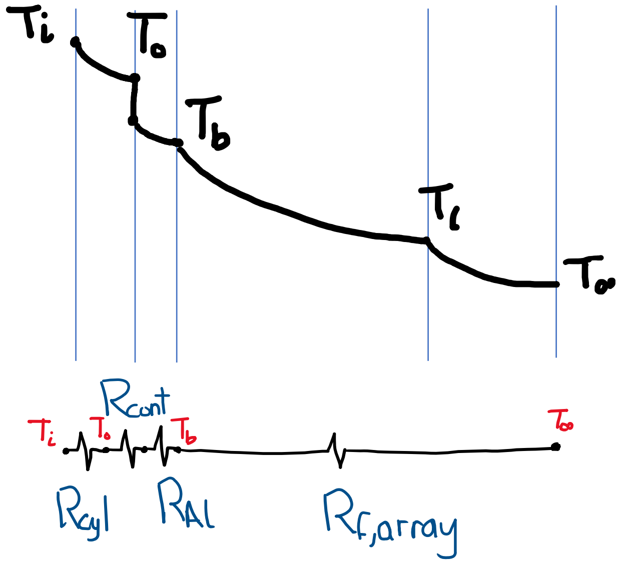
$$L_c^{3/2}(h/kA_p)^{1/2} = 0.616$$

$$r_{2c} = r_2 + \frac{t}{2} = 0.098 + \frac{0.002}{2} = 0.099 \text{ m}$$

$$r_{2c}/r_1 = 0.099/0.066 = 1.5$$

$$\eta_f = 77\%$$

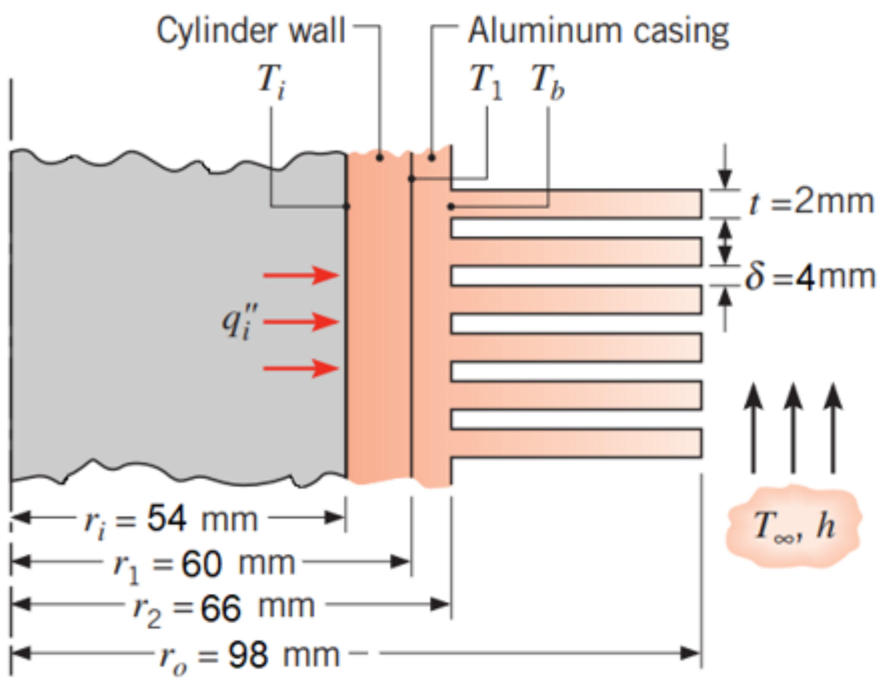




$$\eta_t = 1 + \frac{A_f N_f}{A_t}(\eta_f - 1)$$

$$A_f = (\pi r_o^2 - \pi r_2^2)*2 + 2\pi r_o t$$

$$= 2\pi(0.098^2 - 0.066^2) + 2\pi * 0.098 * 0.002 = 0.0342\ m^2$$



$$A_t = A_{bare} + A_f N_f$$

$$= 2\pi r_2 H - N_f 2\pi r_2 t + A_f N_f$$

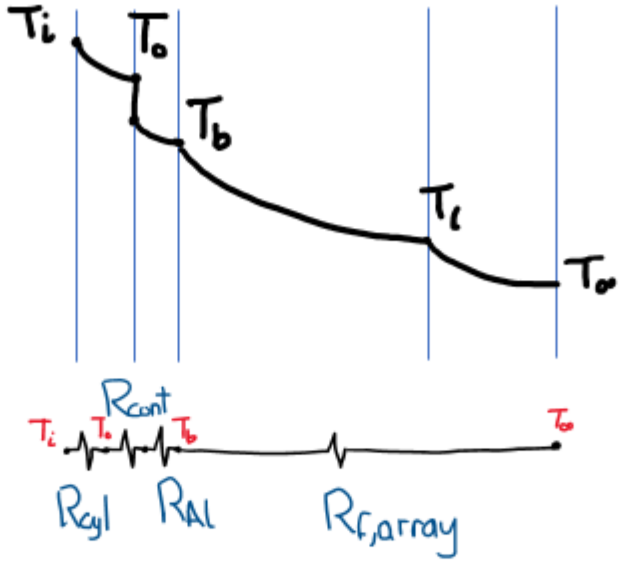
$$= 2\pi * 0.066 * 30 * 0.006 - 30 * 2\pi * 0.066 * 0.002 + 0.0342 * 30$$

$$= 1.0758\ m^2$$

$$\eta_t = 1 + \frac{30 * 0.034}{1.0758}(0.77 - 1) = 0.782$$

$$H = 30 * 0.006$$
$$= 0.18\ m$$

ii) the temp of inner and outer cylinder wall and T_b



$$R_{cyl} = \frac{\ln r_o/r_i}{2\pi H k_{cyl}} = \frac{\ln 60/54}{2\pi * 0.18 * 50} = 0.001863\ K/W$$

$$R_{cont} = \frac{R''_{cont}}{A_1} = \frac{10^{-4}}{2\pi r_1 H} = \frac{10^{-4}}{2\pi * 0.06 * 0.18} = 0.001474\ K/W$$

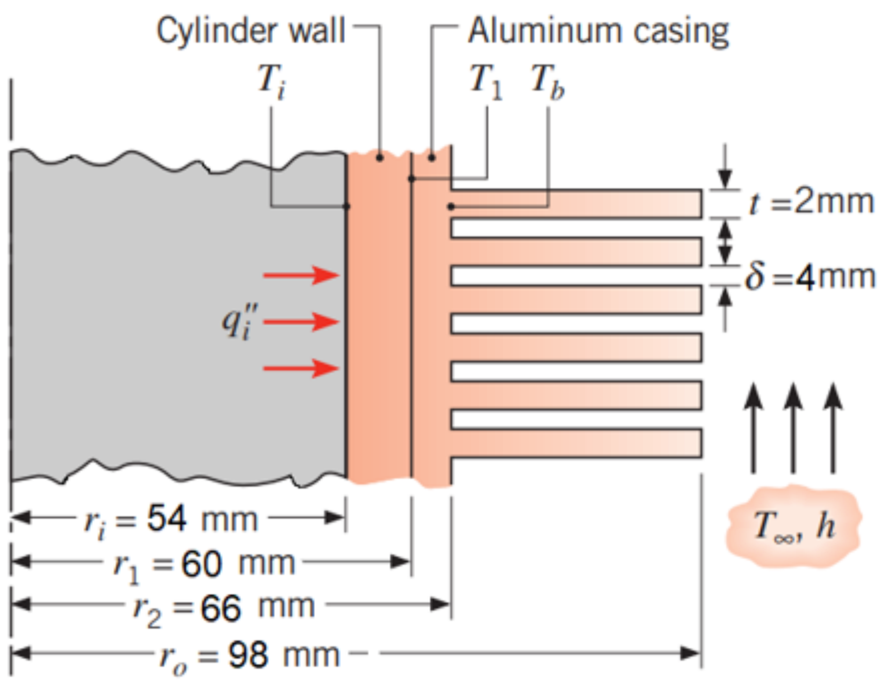
$$R_{Al} = \frac{\ln r_2/r_1}{2\pi H K_{Al}} = \frac{\ln 66/60}{2\pi * 0.18 * 215} = 0.000392 \text{ K/W}$$

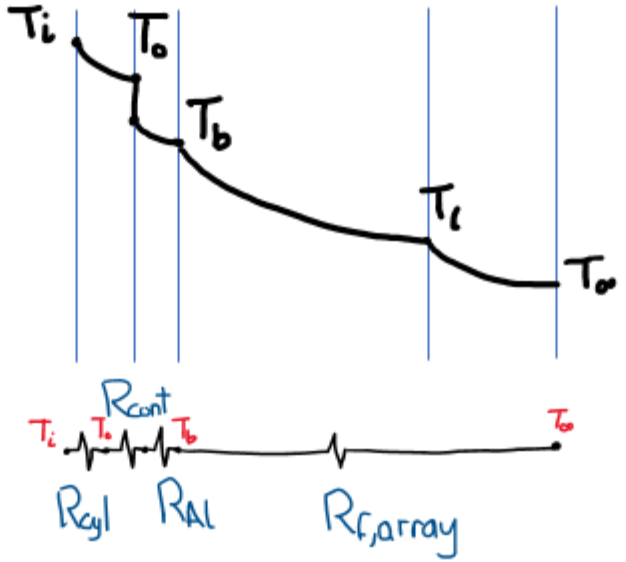
$$R_{f,array} = \frac{1}{\eta_t h A_t} = \frac{1}{0.782 * 150 * 1.0758} = 0.00792 \text{ K/W}$$

$$Q_t = \frac{T_i - T_\infty}{R_{cyl} + R_{cont} + R_{Al} + R_{f,array}} = 7500 \rightarrow T_i = 132.37\,°C$$

$$Q_t = \frac{T_i - T_o}{R_{cyl}} \rightarrow T_o = 121.3\,°C$$

$$Q_t = \frac{T_b - T_\infty}{R_{f,array}} \rightarrow T_b = 104.4\,°C$$





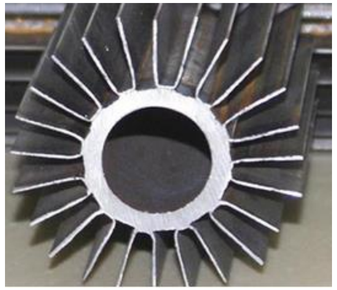
iii) fins effectiveness

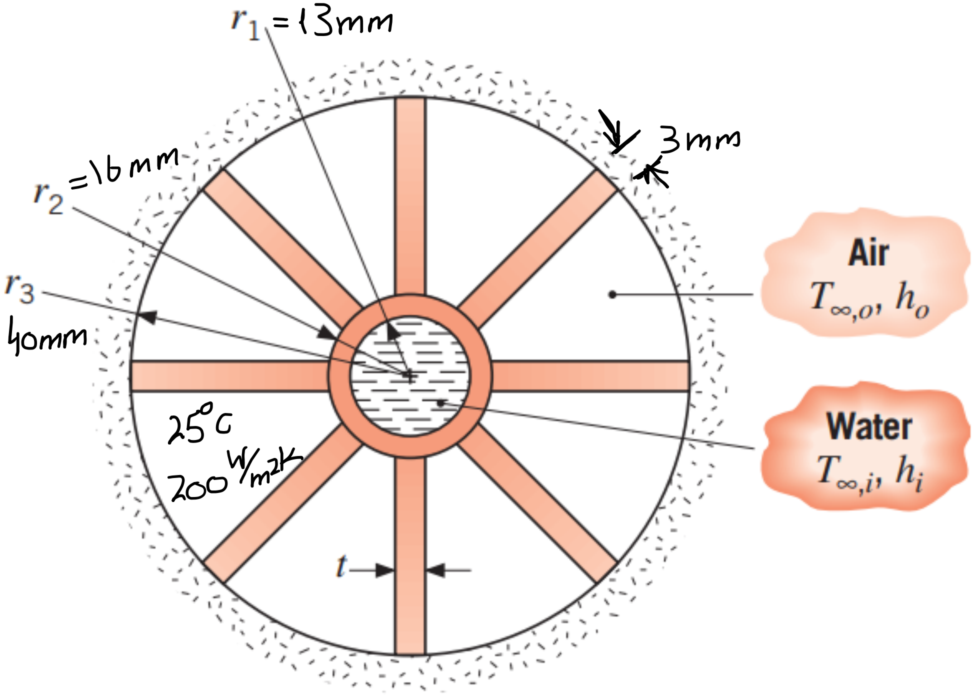
$$\varepsilon_f = \frac{q_f}{q_{w/o}} \quad ; \quad q_f = \eta_f A_f h \theta_b = 0.77 * 0.0342 * 150\,(104.4 - 45)$$
$$= 234.64 \text{ W}$$

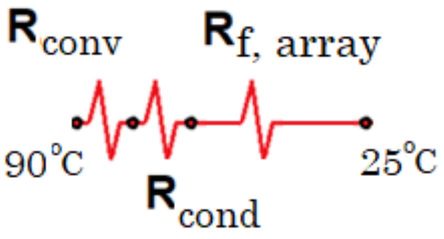
$$q_{w/o} = A_{c,b} h \theta_b = 2\pi * 0.066 * 0.002 * 150(104.4 - 45) = 7.38 \text{ W}$$

$$\varepsilon_f = \frac{234.64}{7.38} = 31.8$$

8.An air heater consists of a steel tube (k= 20 W/m.K), with inner and outer radii are r1=13mm and r2=16mm, respectively and the eight longitudinal fins each of thickness t= 3mm as shown in the figure. The fins extend to concentric tube, which is of radius r3= 40mm and insulated on the outer surface. Water at temperature of T∞= 90°C flows through the inner tube while air at T∞2= 25°C flows through the annular space formed by the larger concentric tube. If hi= 5000W/m2 .K for water and ho= 200 W/m2 .K for air, what is the heat transfer rate per unit length? Also calculate the percentage increases in heat transfer rate due to the addition these fins.







- Heat transfer rate

$$R_{conv_i} = \frac{1}{A_i h_{water}} = \frac{1}{2\pi r_i H h_w} = 0.00245 \ \frac{Km}{W}$$

$$R_{cond} = \frac{\ln r_o/r_i}{2\pi H K} = \frac{\ln 16/13}{2\pi * 20} = 0.00165 \ \frac{Km}{W}$$

$$R_{f,array} = \frac{1}{\eta_t A_t h}, \quad \eta_t = 1 + \frac{N_f A_f}{A_t}(\eta_f - 1)$$

$$A_f = 2(0.04 - 0.016) * \cancel{H}^{=1} = 0.048 \ m^2$$

$$A_t = 2\pi r_2 H - N_f H t + N_f A_f = 0.4605 \ m^2$$

$$l_c = l = 0.04 - 0.016 = 0.024\ m \rightarrow \text{adiabatic tip}$$

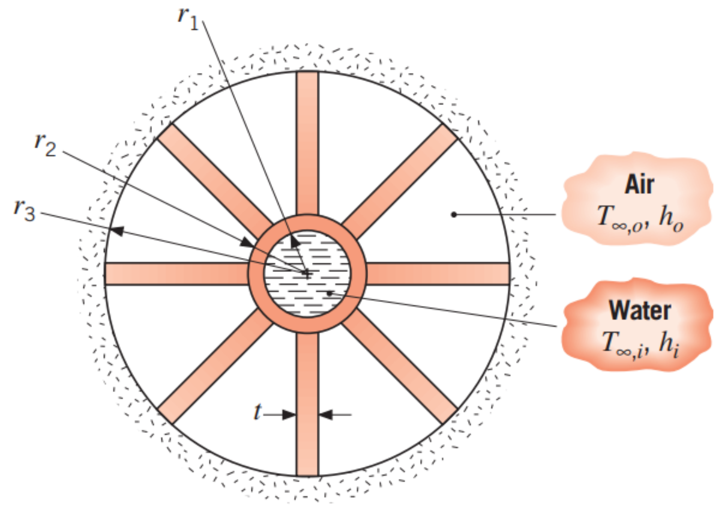
$$l_c^{3/2}\left[h/kA_p\right]^{1/2} = 0.024^{3/2}\left[200/(20 * 0.024 * 0.003)\right]^{1/2} = 1.386$$

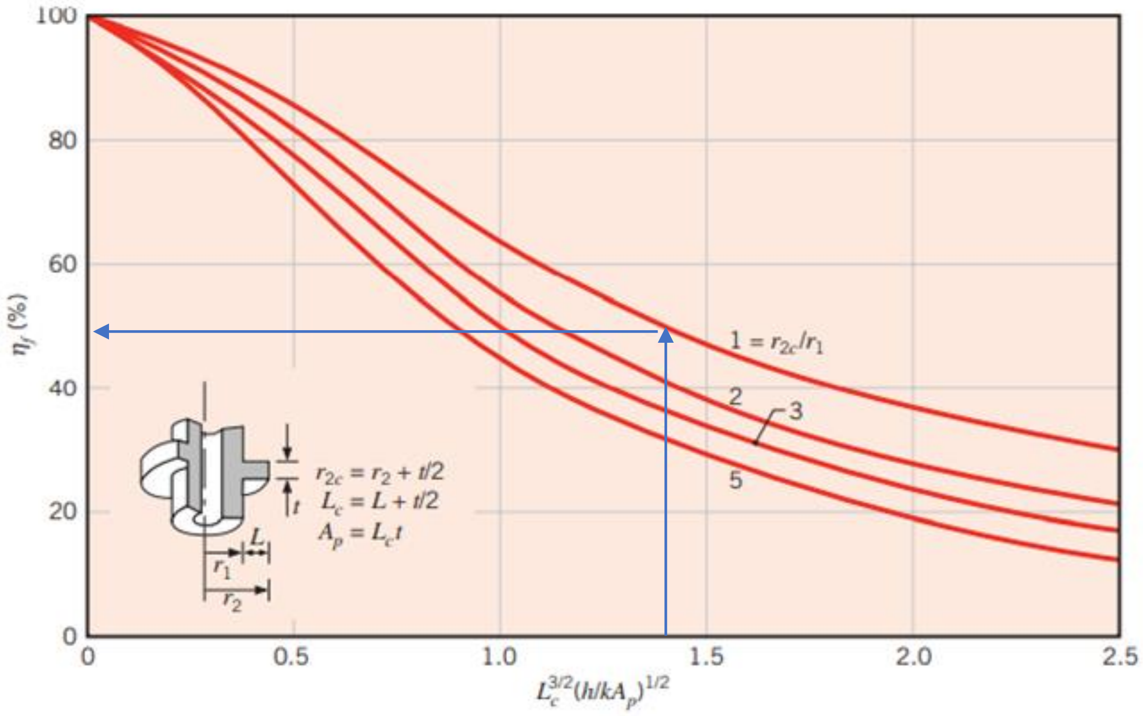
from the graph $\eta_f = 49\%$

$$\eta_t = 1 + \frac{8 * 0.048}{0.4605}(0.49 - 1) = 0.574$$

$$R_{f,array} = \frac{1}{0.574 * 0.4605 * 200} = 0.01889\ \frac{Km}{W}$$

$$q_{w/f} = \frac{90 - 25}{R_{convi} + R_{cond} + R_{f,array}} = 2827\ W/m$$



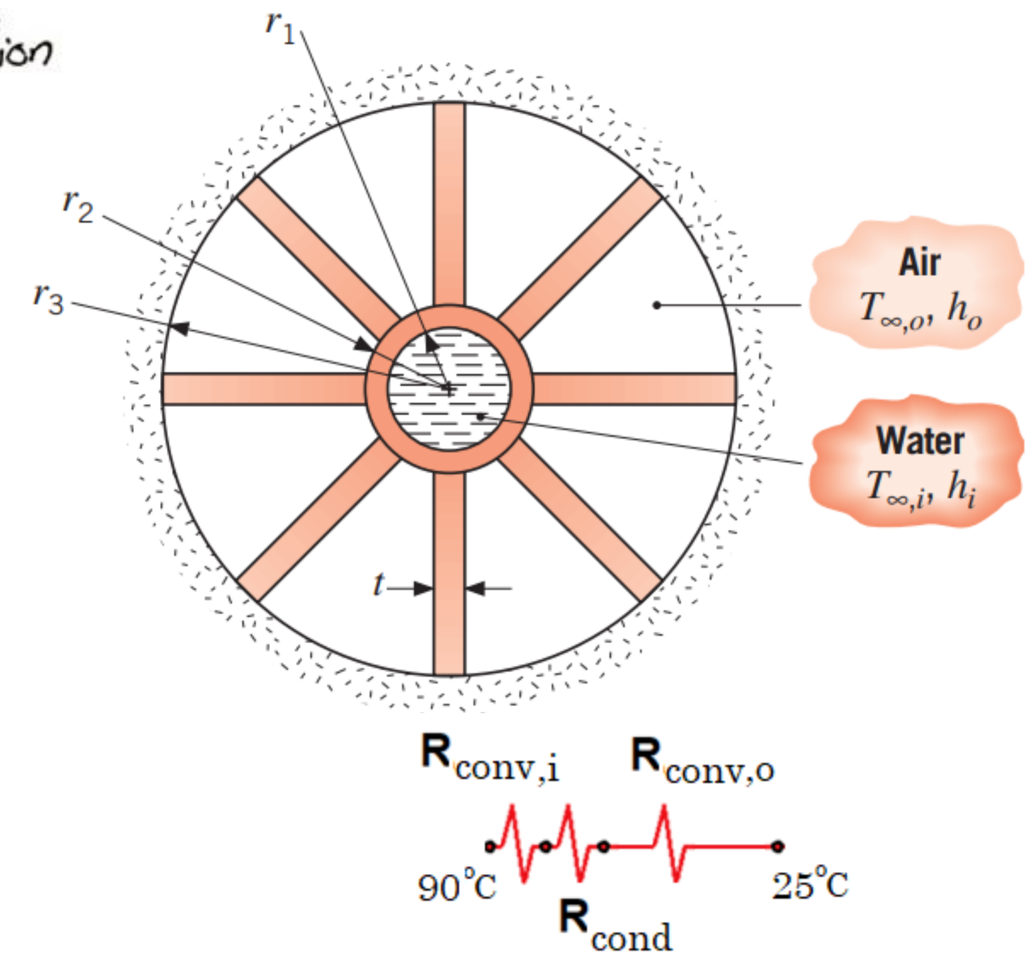


- calculate the percentage increases in heat transfer rate due to fins addition

$$R_{conv_o} = \frac{1}{2\pi r_2 h_o} = \frac{1}{2\pi * 0.016 * 200} = 0.0497 \ W/m$$

$$q_{w/o,fins} = \frac{90 - 25}{R_{conv_i} + R_{cond} + R_{conv_o}} = 1208 \ W/m$$

$$\text{percentage increases in heat transfer} = \frac{q_{w/f} - q_{w/o\,fin}}{q_{w/o\,fin}} = 134\%$$



11. A brass slab is 20mm thick (k=110 W/m.K) has gases on one side T1= 500°C with h1= 15W/m2 .K, while a liquid flows on the other side which T2= 40°C with h2= 2000W/m2 .K. Calculate the rate of heat transfer from the gases to the liquid through the bare slab. It is advised to add brass fins of 1 mm thick, 10 mm long and 10 mm apart on centers to increase the heat transfer rate from gases to liquid. By consider the heat from fin tips, determine where better to add these fins (on gases side or on liquid side)? And calculate the percent increase in the heat transfer rate for the better case chosen.

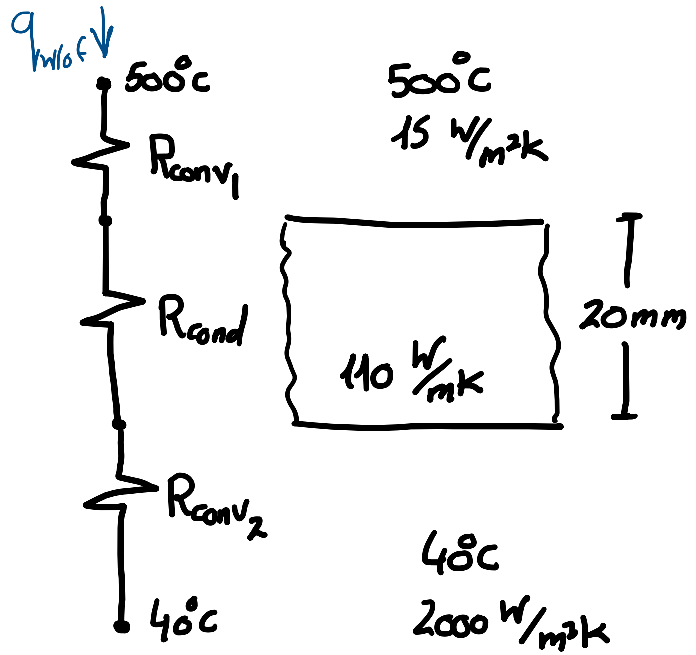
- Calculate the rate of heat transfer from the gases to the liquid

$$R_{conv_1} = \frac{1}{\cancel{A_1} h_1} = \frac{1}{15} \quad \frac{K m^2}{W}$$

$$R_{cond} = \frac{\Delta X}{K} = \frac{0.02}{110} \quad \frac{K m^2}{W}$$

$$R_{conv_2} = \frac{1}{h_2} = \frac{1}{2000} \quad \frac{K m^2}{W}$$

$$q_{w/o\,f} = \frac{500 - 40}{R_{conv_1} + R_{cond} + R_{conv_2}} = 6830 \; W/m^2$$

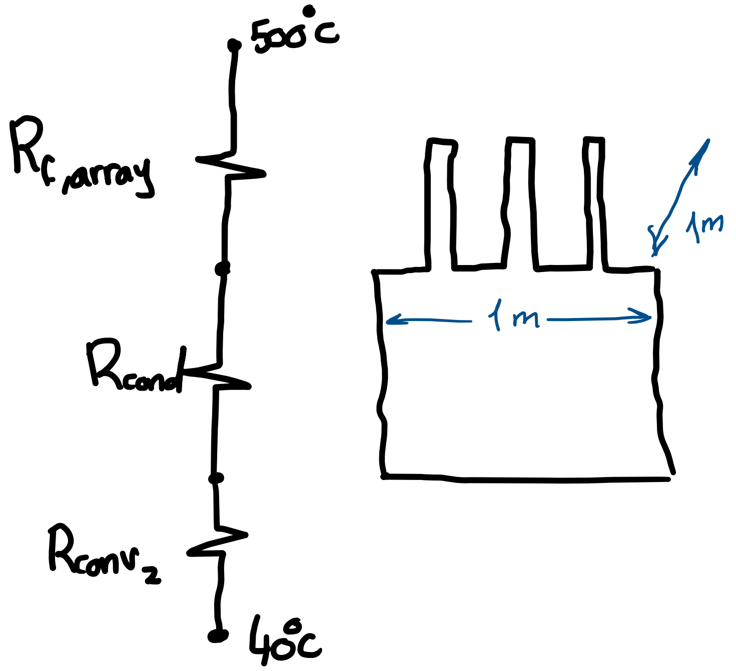


• Determine where better to add these fins

* fins on gas side

$$N_f = \frac{1000}{10} = 100 \text{ fin}$$

$$A_f = p\,l + wt = 2(0.001+1) * 0.01 + 0.001 * 1$$
$$= 0.02102 \ m^2$$



$$A_t = A_{base} + N_f A_f = 1 - N_f wt + N_f A_f$$
$$= 1 - 100 * 1 * 0.001 + 100 * 0.02102 = 3.002 \ m^2$$

Active tip $\quad l_c = l + \frac{t}{2} = 0.01 + \frac{0.001}{2} = 0.0105 \ m$

$$l_c^{3/2} (h/kA_p)^{1/2} = 0.0105^{3/2} (15/(110 * 0.0105 * 0.001))^{1/2} = 0.123$$

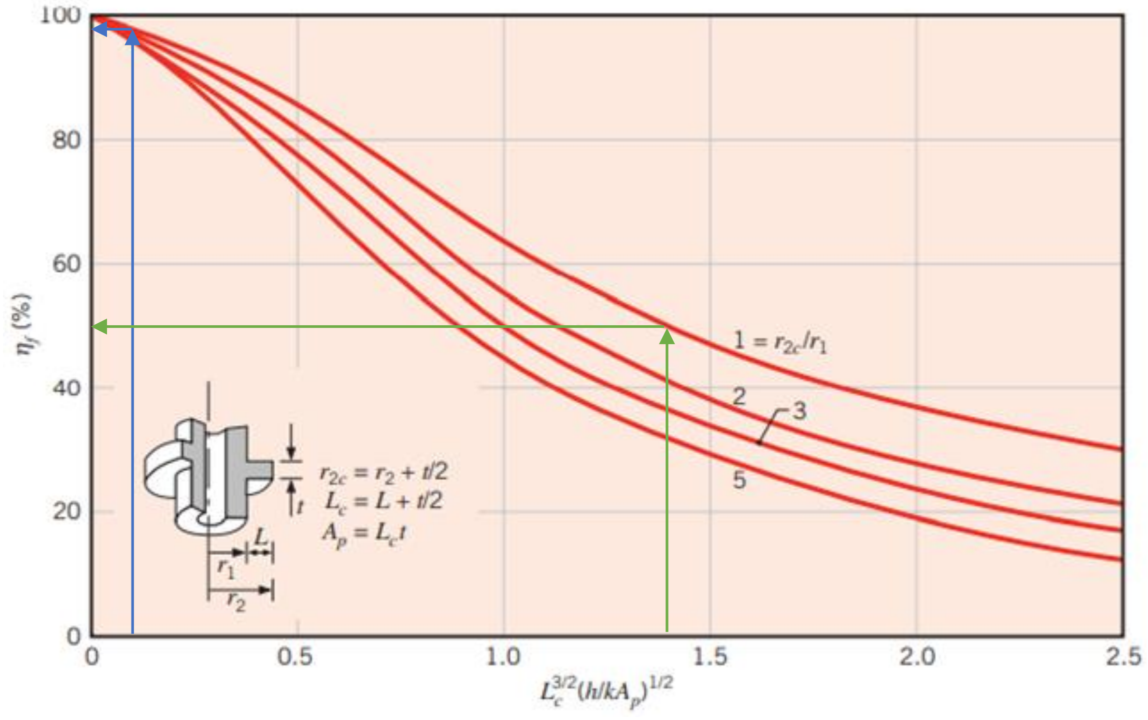
from the graph $\eta_f = 98\%$

$$\eta_t = 1 + \frac{A_f N_f}{A_t}(\eta_f - 1)$$

$$= 1 + \frac{100 * 0.02102}{3.002}(0.98 - 1) = 0.983$$

$$R_{f,array} = \frac{1}{0.983 * 3.002 * 15} = 0.0225 \ \frac{K}{W}$$

$$R_{total} = R_{f,array} + R_{cond} + R_{conv_2} = 0.0232 \ \frac{K}{W}$$



* fins on liquid side

$$l_c^{3/2}(h/kA_p)^{1/2} = 0.0105^{3/2}\left(2000/(110 * 0.0105 * 0.001)\right)^{1/2} = 1.416$$ → graph $\eta_f = 0.5$

$$\eta_t = 1 + \frac{100 * 0.02102}{3.002}(0.5 - 1) = 0.65$$

$$R_{farray} = \frac{1}{0.65 * 3.002 * 2000} = 0.000256 \ \frac{K}{m}$$

$$R_{total} = R_{conv,} + R_{cond} + R_{f,array} = 0.0671 \ \frac{K}{W}$$

it is better to add these fins on gases side where R_{total} is lower

- Calculate the percent increase in the heat transfer rate

$$q_{w/f} = \frac{500-40}{0.0232} = 19828 \ W$$

$$\% \ \frac{q_{w/f} - q_{w/o}}{q_{w/o}} = \frac{19828 - 6830}{6830} = 190\%$$

Thank you