2- The mathematical model relating the changes of an internal combustion engine speed **Δw** with the position of the fuel rack **Δz** is given by:

$$2\frac{dw}{dt} + 0.2\,\Delta w = \Delta z(t)$$

Find:

1- The unit impulse response of the engine

2- The unit step response of the engine

3- The Engine response to the signal shown in fig 2

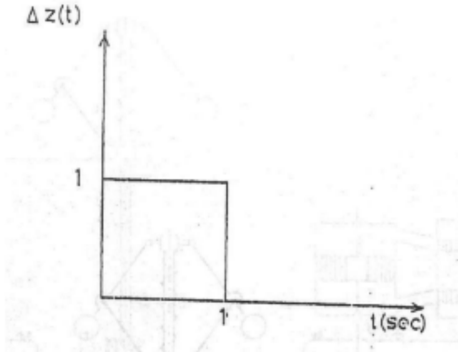

Fig 2

$$\frac{W(s)}{Z(s)} = \frac{1}{2s+0.2} = \frac{5}{10s+1}$$

1) $\frac{5}{10} e^{-t/10}$

2) $5(1-e^{-t/10})$

3) $5(1-e^{-t/10}) - 5(1-e^{-(t-1)/10})\,u(t-1)$

if the mathematical model is given by:

$$2\frac{\mathbf{dw}}{\mathbf{dt}} + \mathbf{0.2}\,\Delta\mathbf{w} = \Delta\mathbf{z}\,(\mathbf{t}-\mathbf{1})$$

Find

4- The unit impulse response of the engine
5- The unit step response of the engine

$$\frac{w(s)}{z(s)} = \frac{5}{10s+1} e^{-(1)s}$$

4) $5/10\, e^{-1/10(t-1)} \quad u(t-1)$

5) $5(1-e^{-1/10(t-1)})\,u(t-1)$

3- Drive the mathematical models relating the head (h) in the accumulator to the input (Q) shown in fig (3). Also, the relation between (Q1) and (Q2) in the transmission system shown in Fig. (4).

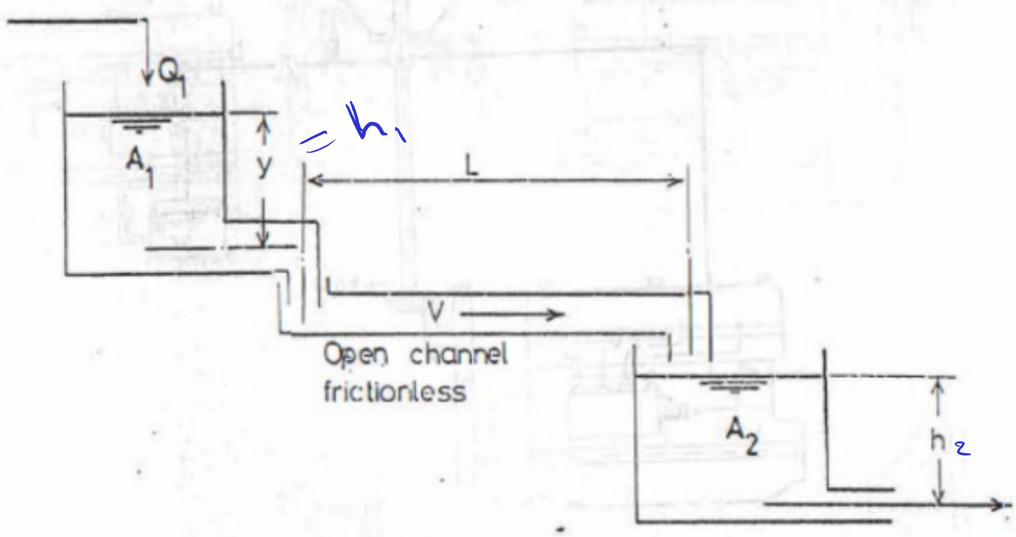

Fig. 3

$$\frac{H_2(s)}{Q_1(s)} = \frac{H_1(s)}{Q_1(s)} \cdot \frac{H_2(s)}{H_1(s)}$$

$$A_1 s H_1(s) = Q_1(s) - C_1 H_1(s)$$

$$\frac{H_1(s)}{Q_1(s)} = \frac{1}{A_1 s + C_1}$$

$$A_2 s H_2(s) = C_1 H_1(s) - C_2 H_2(s)$$

$$\frac{H_2(s)}{H_1(s)} = \frac{C_1}{A_2 s + C_2}$$

$$\frac{H_2(s)}{Q_1(s)} = \frac{C_1}{(A_1 s + C_1)(A_2 s + C_2)}$$

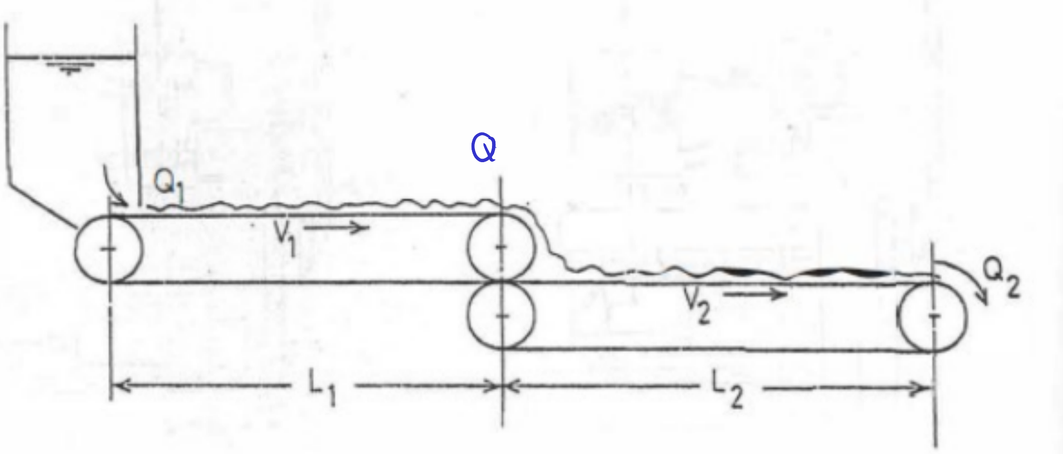

Fig. 4

$$Q(t-\tau_1) = Q(t) \qquad \tau_1 = \frac{L_1}{V_1}$$

$$Q(t-\tau_2) = Q_2(t) \qquad \tau_2 = \frac{L_2}{V_2}$$

4- Fig (5) shows a hydraulic tank with its input and output valves.
Find:
a-The mathematical model relating the discharge qi to the tank and the positions of the input and output valves Δu, Δλ.
b- The mathematical model relating the head y and the signals Δu, Δλ.
c-The impulse and unit-step responses of the tank in cases a and b.

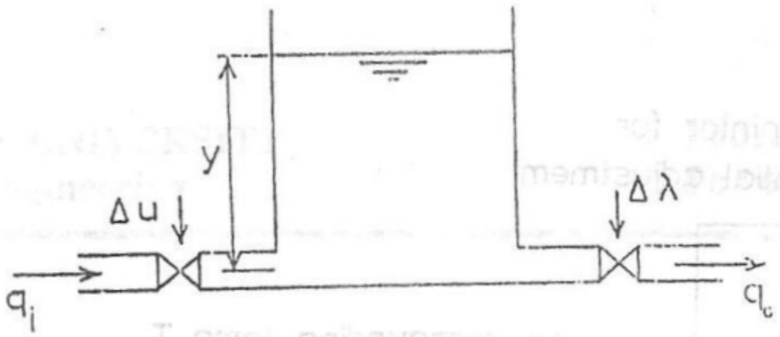

Fig. 5

$$A s Y(s) = C_1 U(s) - C_2 Y(s) - C_3 \lambda(s) + C_4 Y(s)$$

$$Y(s) = \frac{C_1}{A s + \underbrace{C_2 + C_4}_{C_5}} U(s) - \frac{C_3}{A s + \underbrace{C_2 + C_4}_{C_5}} \lambda(s)$$

$$y(t) = C_1/C_5 \left(1 - e^{-\frac{C_5}{A} t}\right) - C_3/C_5 \left(1 - e^{-C_5/A t}\right)$$

5- Fig (6) shows a plant of two hydraulic tanks with a metering pump and the level (h_2) is controlled via the system shown in the Figure. Find:

a- The plant transfer function relating the head (h_2) with the variation of (Q_i) if (Q_3) is constant.

b- Draw the block diagram of the system showing all signals in the Figure and also the transfer functions of the different components.

c- Find the unit- step response of the plant if it has a transfer function in the form

$$W(s) = \frac{10\,e^{-5s}}{s(50s+1)}$$

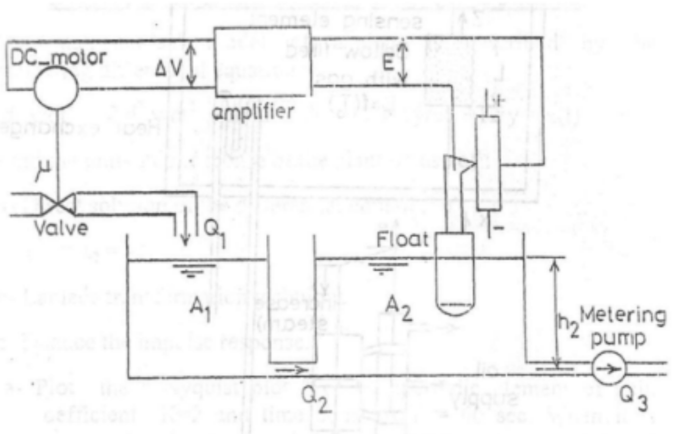

Fig. 6

step: $\dfrac{10}{s^2(50s+1)} = \dfrac{As+B}{s^2} + \dfrac{C}{50s+1}$

$(As+B)(50s+1) + Cs^2 = 10$

$50As^2 + As + 50Bs + B + Cs^2 = 10$

$s^0: \; B = 10$

$s^1: \; As + 50\,\overset{10}{\cancel{B}}\,s = 0 \rightarrow A = -500$

$s^2: \; 50A + C = 0 \rightarrow C = 25000$

$-500 + 10t + 500\,e^{-1/50\,t}$

$\left[-500 + 10(t-5) + 500\,e^{-1/50(t-5)}\right] u(t-5)$

6- Referring to the system shown in Figure 7, determine the values of K and k such that the system has a damping ratio ζ of 0.7 and an undamped natural frequency ω_n of 4 rad/sec.

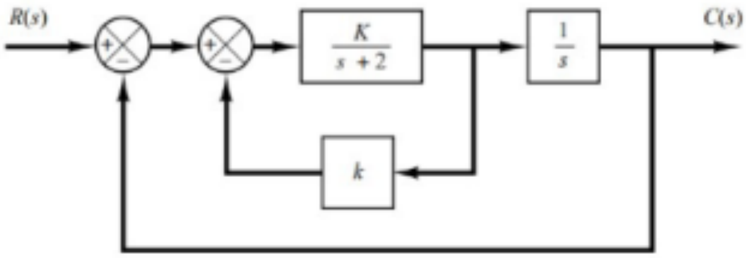

Fig 7.

$$\frac{1}{s} \cdot \frac{\frac{k}{s+2}}{1+\frac{kK}{s+2}} = \frac{K}{s(s+2+kK)}$$

$$\frac{K}{s^2+(2+kK)s+K}$$

$2+kK = 2\zeta\omega_n$ $\qquad K=\omega_n^2$ $\qquad K=16$

$$\frac{2(0.7)(4)-2}{16} = k$$